Spatial navigation with uncertain deviations

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Abstract : We consider geometrical scenes with obstacles and landmarks that can't necessarily be distinguished and generalize the notion of *panoramas* (Sch93; Her94), introduced in the qualitative Spatial Reasoning (QSR) approaches to robot navigation. We study various notions of *motion strategies* in the accessibility graph associated with the local panoramas under uncertain deviations, a natural model of uncertainty for motion planning and navigation. We show that randomized motion strategies can be better than deterministic ones with a finite memory and stress the usefulness of random decisions for qualitative spatial reasoning.

Introduction

Spatial navigation can be viewed as a fundamental task of robotics (Lat90; LL90) and of qualitative reasoning (KL88; Her94). There are many approaches that combine techniques from both areas but one of the main difficulty seems to cope with the uncertainty that most navigation tasks have to address. Dealing with uncertainty is a fundamental issue in robotics where different models such as (DKKN93; Erd90) have been introduced. In the qualitative reasoning area, there are fewer papers dealing with uncertainty and our paper addresses this problem. We consider a model of uncertainty, coined *uncertain deviations* in (BdRS96) but equivalent to many other models such as the ones in (DKKN93), and apply it to the panoramas graphs introduced in the qualitative reasoning area (Sch93; Her94). We define the classical notion of a strategy in a panorama graph and show that random strategies can be better than deterministic ones in this context.

We consider geometrical scenes with obstacles and landmarks that can't necessarily be distinguished and generalize the notion of panoramas introduced in (Sch93). We build the panorama graph, i.e. the graph whose nodes are labeled by a *local* panorama and edges represent possible moves along a line connecting two landmarks and leading from a panorama to another. Some perception models in robotics use similar colored graphs where the color of a node represents the sensors expected measures. In our case, perception is limited to the measure of the sequence of visual local landmarks along a 360 degree rotation. The navigation problem is reduced to traversing the panorama graph under a simple uncertainty model : when instructed to follow an edge e from a point v, we succeed with probability $\mu(e,e)$ but may follow a different edge e_1 adjacent to v with probability $\mu(e,e_1)$. The agent deviates from a plan as a mobile robot deviates from its trajectory. We study how local information, the panoramas, can be used by a moving agent which moves with uncertainty to succeed in its navigation tasks.

We first describe the problem of evaluating the probability of success of plans in the panorama graph : this is a hard combinatorial problem that can be approached by random experiments but the result is known with high probability only. We then define the notion of a strategy : the moving agent knows only the panoramas of the points visited and on this basis decides which edge to follow. The existence and evaluation problems are also hard combinatorial problems for these strategies. We then show how probabilistic strategies, i.e. decision methods allowing random moves can be useful. We show some examples where a random strategy is always better than deterministic strategies with bounded memories.

In the second section we describe the panorama graphs, when scenes have obstacles and when we limit ourselves to local neighborhoods. In the third section we define the uncertainty model. In the fourth section we approach the evaluation problem for plans and in the fifth section the existence and evaluation problems for strategies.

From a geometrical scene to a panorama graph

Let $\mathcal{L}$ be a finite set of distinguishable labels. In a geometrical scene, we isolate points of a set P that are

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possible landmarks with a label $l \in \mathcal{L}$, and a function:

$$L : P \rightarrow \mathcal{L}$$

Two landmarks with the same label can't be distinguished, as two corners in a room or two doors. Consider the standard cellular decomposition associated with P and its panorama-graph as in (Sch93) : nodes represent regions of the cellular decomposition labeled by their panoramas, the sequence of landmark's labels along a 360 degree rotation taking the directions into account. Edges represent possible transitions between adjacent regions. Two cases can be considered, as illustrated by the figures 1 and 2 :

- **The case of global visibility** : all landmarks are visible.
- **The case of local visibility** : only visible landmarks contribute to the cellular decomposition, i.e. for two landmarks A and B, the segment supporting (AB) does not intersect any obstacles.

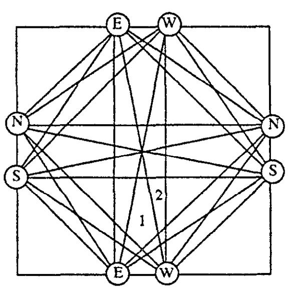

Fig. 1 Global panorama

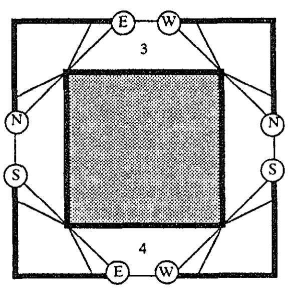

Fig. 2 Local panorama

Example : $\mathcal{L} = \{N, S, W, E\}$. The global panorama at position 1 is :

$$SNwEWeNSsnWewEns$$

We start with the S landmark on the western side and encounter N then w (the southern W landmark from the back, hence written with a small letter), then E and so on. Notice that each landmark occurs twice, once forward with a capital letter and once backwards with a small letter. The panorama at position 1 is a sequence of 16 labels. At position 2, the global panorama is ; $SNEwWeNSsneWwEns$. If all landmarks were distinct, all panoramas would be distinct. Because many landmarks can not be distinguished, some panoramas will be identical.

The local panoramas of the same configuration of landmarks in a room with an obstacle are illustrated in figure 2. Note that although the landmarks are not uniquely identified, it is possible to distinguish the exits by their local panoramas : the position 3 (northern exit in figure 2) is labeled : $EWew$ whereas the position 4 (southern exit in figure 2) is labeled : $WEwe$.

We consider the case of local visibility and the cell decomposition is the geometrical scene with a line extending visible landmarks as long as no obstacles are crossed. Let G be the local panorama graph, i.e. the graph where a point is a region r_i and $D_n = \{r_1, r_2, ...r_n\}$ is the domain of the graph. Nodes of the graph are colored with their panoramas. Let us call $p(r_i)$ the panorama of the point r_i and $\mathcal{P}$ the set all panoramas.

An edge between two adjacent regions is labeled with (A, B), where $A, B \in \mathcal{L}$ if the two regions are adjacent along a segment supported by landmarks p_i and p_j such that $L(p_i) = A$ and $L(p_j) = B$. Let the function *label* defined as

$$label : D_n \times D_n \rightarrow \mathcal{L} \times \mathcal{L}$$

be the partial function which associate the correct labels to the edges. The label function is symmetric in the sense that if $label(r_i, r_j) = (A, B)$, then $label(r_j, r_i) = (B, A)$.

The panorama graph or P-graph is : $G = (D_n, p, label)$ where D_n is the domain, p the panorama function which associates a panorama with every point of the domain, and *label* the previous function.

Example : For the geometrical figure 3 below, the P-graph (without the labels on the nodes and the edges) is shown in figure 4. Notice that the obstacle 's corners behave like landmarks for the local cell decompositions.

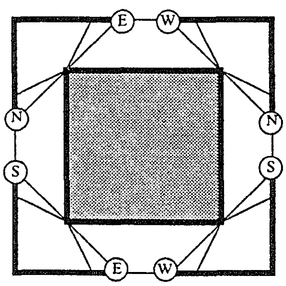

Fig. 3 Square room

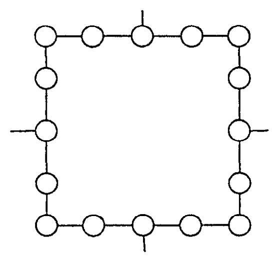

Fig. 4 P-graph

Model of uncertainty

To model the uncertainty, we introduce the deviation function μ on the set of edges E of the P-graph. The set E is the subset of pairs of $D_n \times D_n$ for which the function *label* is defined.

$$\mu : E \times E \rightarrow [0, 1]$$

The function μ is defined for pairs of edges with a common tail. If e and e_1 do not have a common tail, $\mu(e, e_1) = 0$. Let $e = (v, w)$ be an edge to follow. The motion will be along the edge $e_1 = (v, u)$ with the probability $\mu(e, e_1)$, and the vertex w will be reached only with the probability $\mu(e, e)$. We assume that

$$\sum_{e_1 \in OUT(v)} \mu(e, e_1) = 1, \qquad (1)$$

i.e. that one of the possible movements takes place ($OUT(v)$ is the set of edges whose tail is v). The probability space is the set of nodes adjacent to v and we consider the probability distribution μ as given.

This model of uncertainty is natural for navigation and motion planning : a moving agent always deviates from an original route and one can reasonably say that the deviations are probabilistic. The colorings of the nodes model the points that can be distinguished by the panoramas. In a physical model, the sensors would give the panoramas (possibly with some approximation).

The evaluation of plans

Let G be a P-graph, μ the uncertainty function and two points $s, t \in D_n$. We wish to navigate from s to reach t. Given $(D_n, p, label, \mu, s, t)$, a plan is a path connecting s to t in G.

Definition 1 *A plan $\pi = < l_1, ..., l_k >$ is a sequence of edge labels connecting s to t. A possible path σ is a sequence $s = v_0, v_1, \ldots, v_i, ..., v_k$ such that $\mu((v_{i-1}, v_i), e_i) \neq 0$ where e_i is the edge whose label is l_i.*

A possible path is simply the execution of a plan under the probabilistic model. The probability associated with a possible path σ and a plan π is

$$\mathbb{P}rob(\sigma \,/\, \pi) = \prod_{i=1}^{k} \mathbb{P}rob(((v_{i-1}, v_i) \,/\, l_i) = \mu((v_{i-1}, v_i), e_i)$$

if e_i is the edge whose label is l_i. Let $CONE(\pi)$ the set of possible paths. The probability to reach t starting in s is the probability that π succeeds.

$$\mathbb{P}rob[\pi \; succeeds] = \sum_{\sigma \in CONE(\pi)} x_\sigma \,.\, \mathbb{P}rob(\sigma, \pi)$$

where $x_\sigma = 1$ if σ reaches t and 0 otherwise.

Given a plan, we can ask how to evaluate its reliability, i.e. the probability it will succeed. We deal with the decision problem :

Evaluation of plans in the presence of uncertainty :

input: G_n the panorama graph, μ the uncertainty function, a plan π and a rational q

output: 1 if $\boldsymbol{Prob}(\pi) > q$, 0 otherwise.

This problem is the decision problem associated with a classical $\#P$-problem, called $s - t$-PATHS. It consists in counting the number of simple paths in a graph (see (Val79)), and can't be solved in polynomial time if we follow the standard hypothesis in complexity theory. We may then study special cases where this problem can be computed in polynomial time (MdRR94) or directly deal with the more general notion of a strategy.

Motion strategies

In the previous plans, there is a very limited spatial reasoning, as the plan is fixed at the beginning. A strategy generalizes the notion of a plan and perception is taken into account in order to cope with the uncertainty. The perception model is that the moving agent only knows the panoramas along the nodes of possible paths. It must use this qualitative information to navigate. Let $\mathcal{P}$ be the set of distinct panoramas.

Definition 2 *A strategy is a function σ which assigns to a finite sequence of panoramas (the history of the perception of the visited points) an edge label describing uniquely the edge to follow.*

$$\sigma : \mathcal{P}^* \rightarrow \mathcal{L} \times \mathcal{L}$$

If $\sigma(p_{i_1}, ...p_{i_m}) = (A, B)$, we interpret the situation as: starting in s with panorama p_{i_1} we followed $m-1$ transitions, reached a point whose panorama is p_{i_m}, and decided to take the edge labeled (A, B). The *semantics* of a strategy σ (or the *behavior* due to σ) is given by the random mapping $path_\sigma : \boldsymbol{N} \rightarrow D^*$ which for every $k \in \boldsymbol{N}$ defines a random possible path following σ after k steps. The motion starts from s and on the basis of $p(s)$, σ chooses some edge e and with the probability $\mu(e, e_1)$ follows an edge e_1 and so on.

Examples of strategies

- σ_0 : *Find a shortest path, then follow it whenever it is possible*

  This strategy is like a plan, except we first compute it. It will stop at the first difference between the expected panorama and the one observed.

- σ_1 : *Find a shortest path, then follow it as long as the 10 consecutive panoramas are different. Otherwise declared yourself lost.*

  This strategy may stop very early on (as the previous one) as the computed edge label may not exist on a given position.

- σ_2 : *If the panorama is AaBb follow the edge (A, B). If the previous panorama was AaBb and the current one is BaAb then follow the edge (B, A).*

Quality criteria of strategies

The general problem is to reach t from s as fast as possible with the maximal probability, or more generally, with the maximal gain. This motivates the following **Reliability** criteria :

Within a given time (number of steps) we wish to reach t from s with the maximal possible probability. Let us define :

- $\boldsymbol{R}(\sigma, k) = \boldsymbol{Prob}(\sigma$ leads from s to t in not more than k steps),
- $\boldsymbol{R}_\infty(\sigma) = \boldsymbol{R}(\sigma) = \sup_k \boldsymbol{R}(\sigma, k)$.

It is clear that $\boldsymbol{R}(\sigma, k) = \boldsymbol{Prob}[$there exists a simple realization of σ of length not greater than k that leads from s to $t]$. As different simple realizations mentioned above constitute incompatible events, $\boldsymbol{R}(\sigma, k)$ is the sum of probabilities of the mentioned simple realizations. More formally, let $SPATHS(v, w, k)$ denote the set of all simple paths of length not greater than k between v and w in G. Then

$$\boldsymbol{R}(\sigma, k) = \sum_{\pi \in SPATHS(s,t,k)} \boldsymbol{Prob}(\pi \text{ is a realization of } \sigma) \tag{2}$$

The case of distinguishable landmarks

Assume that the landmarks include all the obstacles corners that occur in the local decomposition. If it is not the case, consider them as non-distinguishable landmarks, the case studied in the next subsection. If all the landmarks are distinct, a given panorama identifies a node in the P-graph only in the case of global panoramas. In this case, there is always a simple strategy which succeeds with reliability ($\boldsymbol{R}_\infty$) 1 (assuming the P-graph is connected).

Theorem 1 *If the labeling function L is bijective, there exists a strategy which for any $s, t \in D_n$ succeeds with probability 1 in the P-graph of global panoramas.*

proof : Every panorama identifies one unique node of the graph. At any point v, we can compute a shortest path from v to t and follow it as a plan. With positive probability, we get closer to t. Hence after enough time, we will reach t. □

Notice that there is no bound on the time necessary to reach t. In the case of local panoramas, we can not distinguish the nodes of the graph by their panoramas and we deal with the general case considered in the next section.

The case of non-distinguishable landmarks

In most cases occurring in robotics, landmarks are not uniquely identifiable and the problem of existence of reliable strategies is open. For a given strategy, we can also ask to evaluate its probability of success, and more generally for the construction of reliable strategies. In the sequel, we consider the **Existence** and **Evaluation** problems only.

The problem of existence of a strategy

Given *a panorama graph, two points $s, t \in D_n$, a rational q* decide *whether there exists a strategy with $\boldsymbol{R}_\infty > q$.*

The problem of Evaluation of a strategy

Given *a strategy σ*, compute $\boldsymbol{R}_\infty(\sigma)$.

In (BdRS96), it is shown that these problems are hard in some specific complexity theoretic sense. It is then natural to look at randomized algorithms for these problems. Let us first consider randomized (or probabilistic) motion strategies in this context.

Deterministic vs. probabilistic

In many problems with uncertainty, probabilistic strategies are very useful. They consist in simply allowing a random decision at some particular points, i.e. flipping a coin. Formally, a probabilistic strategy is a function:

$$\sigma : \mathcal{P}^*.\{0,1\}^* \rightarrow \mathcal{L}.\mathcal{L}$$

In addition to the previous input, we read a sequence of random bits (0 or 1) which will determine the random decisions we will take.

Examples of probabilistic strategies Consider a generalization of the previous scene, where we assemble similar scenes identical to the figure 3. In order to traverse the graph and reach the target t, we need to cross few layers of identical obstacles.

- σ_{p_1} : *Make a random walk*
- σ_{p_2} : *Follow the strategy σ_1. When lost make a random walk*

Notice that σ_{p_1} is purely random, whereas σ_{p_2} first starts with a deterministic strategy and switches to a random strategy when a criterium is true (being lost).

Probabilistic strategies can be better We want to construct an artificial scene for which a probabilistic strategy (in fact, the random walk σ_{p_1}) is provably better than any deterministic strategy with a finite memory. The scene is a maze built as a grid whose basic element is the scene of figure 3, whose P-graph is given in figure 4. We need a specific central room given in the figure 5, whose P-graph is given in the figure 6 below. Navigation will always start at the position 1 of the central room.

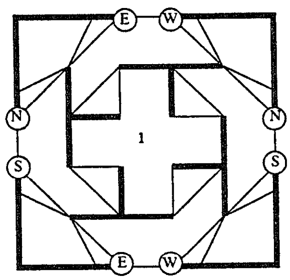

Fig. 5 Central room

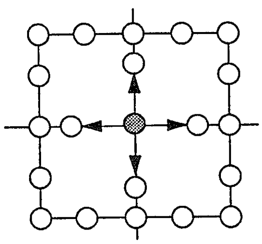

Fig. 6 P-graph

The global scene is the maze given in the figure 7, whose P-graph (the central part) is given in the figure 8. It is made of $n^2 - 1$ square rooms connected to the central room.

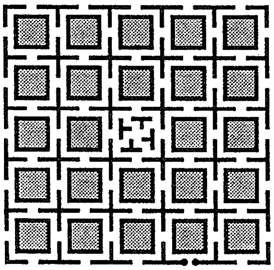

Fig. 7 Maze

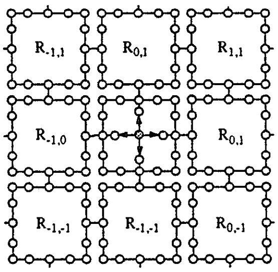

Fig. 8 P-graph (central part)

The P-graph : There are two distinguished landmarks, the center s and the target t, located on the southern border (4th gate from the left). All the other landmarks created by the obstacle corners are not distinguishable. Consequently all panoramas are identical except for the central position and the points close to the targets.

The uncertainty model : All edges are certain except for the center c where the uncertainty is uniformly distributed, i.e. for all edges e_c, e'_c leaving c :

$$\mu(e_c, e'_c) = 1/4$$

To simplify the analysis, consider the maze K_n as a regular (n, n) grid as shown on the figure 9 below. The center s of K_n is the point of coordinate $(n/2, n/2)$. Each point of the grid has four outgoing edges, labeled N, W, E, S. The edges connect to the natural North, West, East and South Neighbors, except on the limits of the grid where they connect to themselves. For example the North edge of the upper left corner of the grid connects to itself.

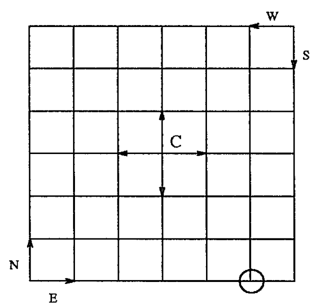

Fig. 9 The simplified grid

Lemma 1 *For any deterministic strategy with finite memory M,*

$$\mathbb{P}rob[\text{target is reached}] \leq 1/4$$

proof : All the points are indistinguishable except for the target point. A deterministic strategy is a function which associates with the memory content a decision (N, W, S or E) and a new memory content. There are finitely many such functions and after at most N iterations we will find two points a and a', covered by the strategy, such that the memory contents will be identical : hence the decisions on these two points and all the consecutive points thereafter will also be identical.

The trace of the strategy will enter a loop and will evolve in one of the four directions, depending on the positions of a and a'. If a' is south of a, the strategy will evolve towards the South until it reaches the southern limit (see figure 10).

If the target is placed randomly on one of the four limits (north, south, east and west), then :

$$\mathbb{P}rob[\text{target is reached}] \leq 1/4$$

□

In actual fact, the observation above can be refined as the space spanned by the strategy is much smaller than the limit size. The figure 10 shows a realization of a deterministic strategy that evolves towards the south and will reach the limit after having passed through a, a' and a''. It is successful as it reaches the target.

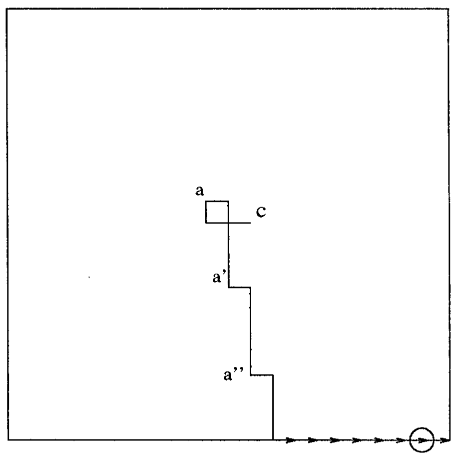

Fig. 10 The trace of a finite memory strategy.

The key point of this construction is the initial uncertainty which with probability $1/4$ puts us in one of the four quadrants. Nothing can allow us to realize which of the quadrant we ended up in and the probability of success can't exceed $1/4$. A classical result of (AKR$^+$79) is : *A random walk on the grid K_n will reach the target with probability greater than $1/2$ after $O(n^3)$ iterations.*

On can conclude that on this simple scene, the random walk can outperform any deterministic strategy with finite memory and obtain:

Theorem 2 *On the simplified grids defined in the figures 7,8 and 9, the random walk outperforms any deterministic strategy with a finite memory.*

Conclusion

We defined a spatial model that allows for non distinguishable landmarks based on labeled panoramas. It is a labeled graph that defines the accessibility relations when we deal with landmarks and obstacles. We then apply the model of uncertain deviations for motion strategies in this model. We showed that randomized strategies can outperform deterministic ones on simple grids and we believe that randomized algorithms are important in the context of spatial reasoning with uncertainty.

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