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Some Variations on Default Logic

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Abstract

In the following paper, we view applying default reasoning as a construction of an argument supporting agent's beliefs. This yields a slight reformulation of the notion of an extension for default theories. The proposed formalism enjoys a property which we call rational maximization of beliefs.

Introduction

The fundamental importance of default reasoning in AI had been recognized long before appeared its first formalization (Reiter 1980). A default reasoning agent is able to derive her conclusions using the following inference patterns: *if there is no reason to believe something else, assume that...* This enables to fill up the gaps in agent's knowledge that result from incomplete information and unblock construction of arguments that could support her beliefs. Such patterns of inference allow to represent exceptions together with general knowledge about what things normally can be expected to be like. Since any default conclusion might be subject to change when new information is provided, default reasoning is nonmonotonic.

Default logic is defined by extending the language of some base standard logic[1] such as predicate logic (either classical or modal) by specific inference rules, called *defaults*.

Definition 1 Any *default* δ has the form:

$$[\alpha : \beta_1, \ldots, \beta_n / \gamma]$$

where α, β_i, and γ are formulae of the base logic and are called the *prerequisite, justification,* and *consequent* of δ, respectively.

Definition 2 If δ is a default, then

$$\begin{aligned} pre(\delta) &= \alpha, \\ jus(\delta) &= \{\beta_1, \ldots, \beta_n\}, \\ con(\delta) &= \gamma. \end{aligned}$$

[1]In the following discussion we will use propositional calculus. Applying a technique given in (Reiter 1980) we can easily generalize our results into more interesting cases in which some or all of the formulae appearing in defaults may contain free variables.

If D is a set of defaults, then

$$\begin{aligned} PRE(D) &= \{pre(\delta) : \delta \in D\}, \\ JUS(D) &= \bigcup_{\delta \in D} jus(\delta), \\ CON(D) &= \{con(\delta) : \delta \in D\}. \end{aligned}$$

In certain circumstances, defaults allow to augment the set of agent's beliefs that follow deductively from what she knows about the world. Assumption of a consequent of some default is conditioned by the requirement that every justification of this default is consistent with the belief set. Therefore, the meaning of defaults depends on the belief set of the agent.

The agent's knowledge might be represented by a pair $\Delta = (W, D)$, called a *default theory*, where W and D are sets of formulae (axioms) and defaults, respectively. Δ delimits the agent's belief set, which, since defaults refer to it, must be defined as a fixed-point construct over Δ. Such a belief set is generally called an *extension* of Δ. Here is the simplest definition of this notion.

Definition 3 (Reiter 1980) Let $\Delta = (W, D)$ be a default theory. For any set S of formulae, define $\Gamma_\Delta(S)$ as the minimal set satisfying the following conditions:

- $W \subseteq \Gamma_\Delta(S)$,
- $Th(\Gamma_\Delta(S)) = \Gamma_\Delta(S)$,
- if $[\alpha : \beta_1, \ldots, \beta_n / \gamma] \in D$, $\alpha \in \Gamma_\Delta(S)$ and $\neg\beta_1, \ldots, \neg\beta_n \notin S$, then $\gamma \in \Gamma_\Delta(S)$.

We say that E is an *a-extension* of Δ if and only if $E = \Gamma_\Delta(E)$.

Unfortunately, some default theories may have no a-extension. There are known large classes of default theories for which the existence of an a-extension is guaranteed (Reiter 1980, Etherington 1986). Nevertheless, they might turn to be inadequate to represent knowledge in some specific cases. Nonexistence of an a-extension of some theories steams from the fact that the application of defaults is not sufficiently constrained: a situation is possible in which a justification of an applied default is denied by axioms and consequents of some subset of all applied defaults.

Łukaszewicz (1988) modifies the notion of an extension imposing new applicability criteria.

Definition 4 (Łukaszewicz 1988) Let $\Delta = (W, D)$ be a default theory. For any two sets S and T of formulae, define $\Gamma^1_\Delta(S,T)$ and $\Gamma^2_\Delta(S,T)$ as the minimal sets satisfying the following conditions:

- $W \subseteq \Gamma^1_\Delta(S,T)$,
- $Th(\Gamma^1_\Delta(S,T)) = \Gamma^1_\Delta(S,T)$,
- if $[\alpha : \beta_1, \ldots, \beta_n/\gamma] \in D$, $\alpha \in \Gamma^1_\Delta(S,T)$ and for all $\varphi \in T \cup \{\beta_1, \ldots, \beta_n\}$, $S \cup \{\gamma\} \not\vdash \neg\varphi$, then
  - $\gamma \in \Gamma^1_\Delta(S,T)$,
  - $\beta_1, \ldots, \beta_n \in \Gamma^2_\Delta(S,T)$.

We say that E is a *b-extension* of Δ with respect to F if and only if $E = \Gamma^1_\Delta(E,F)$ and $F = \Gamma^2_\Delta(E,F)$.

This modification causes that every default theory has a b-extension. Moreover, the new formalism has the property of *semimonotonicity*, which means that introducing new defaults into a default theory does not change any of the previously derivable beliefs.

In the following discussion, we will suggest and examine further changes to the notion of an extension.

Defaults as arguments[2]

We claim that the definitions given in the previous section do not reflect the way in which some perfectly rational agent would explain her own beliefs.

Suppose that all what the agent knows about the world can be expressed by a simple default theory

$$(\emptyset, \{d_1 = [\top : r/p], d_2 = [\top : s/p], d_3 = [p : t/t]\}) \quad (1)$$

where $\top$ stands for a tautologically true formula. According to Definition 3 (or Definition 4), the only extension of (1) is $Th(\{p,t\})$. Let us observe that these definitions force every default of (1) to be applied. A perfectly reasoning agent, however, would notice that in order to explain t, it is sufficient to refer to d_3 and one of the remaining defaults, either d_1 or d_2. Referring to both d_1 and d_2 is somewhat redundant. Therefore, there are two possible explanations why the agent believes t. She can use the following arguments: *Since I find r to be consistent with what I believe then, by d_1, I assume that p holds and hence, by d_3, I can conclude t.* She uses defaults d_1 and d_3 (in this order) as arguments justifying her belief in t. Of course, she can use d_2 and d_3 (in this order) to achieve the same aim. But in both cases, she realizes that it is superficial to consider a default and think whether its justification is consistent with her beliefs, if she already knows its consequent.

This kind of reasoning seems to be manifested when we attempt to explain something in the most concise way. In such case, we try to use, for the sake of clarity, only those arguments that are necessary to achieve our task. Łukaszewicz (1988) maintains that it is a worse approximation of human reasoning than the one proposed in Definition 4. However, although people are not logically omniscient, they seem to frequently use such a pattern of inference, at least in simple cases. Nevertheless, this kind of reasoning certainly can be attributed to perfect reasoners.

Summarizing the above discussion, the perfectly reasoning agent treats defaults as arguments choosing only those defaults whose consequents cause the progress of the process of generating her own beliefs.

The question remains, how this new constraint imposed on applicability criteria for defaults changes the notion of an extension. Not surprisingly, this new constraint does not change a-extensions. Let us recall that the only condition that must be satisfied by a default in order to include its consequent into an a-extension, is that its justifications are consistent with some current set of beliefs. It does not matter, therefore, if we use all applicable defaults or only the "strongest", whose consequents together with axioms entail the consequents of all other potentially applicable defaults. Of course, it might happen that there are no "strongest" defaults, but if they exist, then in both cases, we would generate the same set of a-extensions. On the other hand, b-extensions are vulnerable to this new policy of applying defaults. From the definition of a b-extension, it follows that applying a default we have to check not only whether all of its justifications are consistent with some current set of beliefs, but additionally, if its consequent does not interfere with the justifications of the previously applied defaults. It might happen, therefore, that restricting the application of defaults only to the "strongest" unblocks some other defaults which would otherwise remain inapplicable.

Let us formalize our intuitions concerning the rationally thinking agents and redefine the notion of an extension for default theories.

Definition 5 By an *indexed formula* we will understand a pair $\langle\alpha, \delta\rangle$ where α is a formula and δ is a default or a special symbol ε.

Definition 6 If S is a set of indexed formulae, then

$$\begin{aligned}\mathcal{K}(S) &= \{\langle\alpha, \varepsilon\rangle \in S\},\\ \mathcal{F}(S) &= \{\alpha : \langle\alpha, \delta\rangle \in S\},\\ \mathcal{D}(S) &= \{\delta \neq \varepsilon : \langle\alpha, \delta\rangle \in S\}.\end{aligned}$$

The next two definitions formalize what we will understand by a "stronger" default.

Definition 7 We say that an indexed formula $\langle\alpha, \delta\rangle$ is *weakly subsumed* by a set S of indexed formulae if and only if there is $S' \subseteq S$ such that $\langle\alpha, \delta\rangle \notin S'$, $\mathcal{F}(S') \vdash \alpha$ and for every default $\xi \in \mathcal{D}(S')$ there is a sequence $S_0, \ldots, S_n \subseteq S$ satisfying the following conditions:

(*a*) for every $0 \leq i \leq n$, $\langle\alpha, \delta\rangle \notin S_i$,

(*b*) $\mathcal{F}(S_0) \vdash pre(\xi)$,

(*c*) for every $1 \leq i \leq n$, $\mathcal{F}(S_i) \vdash PRE(\mathcal{D}(S_{i-1}))$,

[2]The proofs of all cited theorems can be found in (Rychlik 1991).

(d) $S_n = \emptyset$.

To illustrate the above notion, let us consider two sets of indexed formulae:

$$S' = \{s_1, s_2, s_3\} \quad \text{and} \quad S'' = \{s_1, s_2, s_3, s_4\}$$

where

$$\begin{aligned} s_1 &= \langle q, [\top : p/q] \rangle, \\ s_2 &= \langle r, [q : r/r] \rangle, \\ s_3 &= \langle q \wedge s, [r : s/q \wedge s] \rangle, \\ s_4 &= \langle q \wedge t, [\top : t/q \wedge t] \rangle. \end{aligned}$$

Let us note that s_1 is not weakly subsumed by S'. Although we have that $\mathcal{F}(\{s_3\}) \vdash q$ and $\mathcal{F}(\{s_2, s_3\}) \vdash q$, the only sequence satisfying conditions (b)–(d) for the default of s_3, namely the sequence $\emptyset, \{s_1\}, \{s_2\}$, fails to satisfy condition (a). On the other hand, s_1 is weakly subsumed by S'', because $S'' \supseteq \{s_4\} \vdash q$ and, for the default of s_4, the sequence consisting of only one empty set of indexed formulae satisfies conditions (a)–(d).

Definition 8 We say that an indexed formula ς is *subsumed* by a set S of indexed formulae if and only if there is $S' \subseteq S$ such that ς is weakly subsumed by S' and no element of S' is weakly subsumed by S.

Of course, if an indexed formula is subsumed by a set of indexed formulae, then it is also weakly subsumed by this set. However, the converse is not true. As an example, let us consider the following infinite set of indexed formulae:

$$S = \{s_0, s_1, \ldots\}$$

where $s_i = \langle q_0 \wedge \ldots \wedge q_i, [\top : p/q_0 \wedge \ldots \wedge q_i] \rangle$, for all $i \geq 0$. It is easy to see that for every $s_i \in S$, $\{s_{i+1}\}$ satisfies conditions (a)–(d), which means that s_i is weakly subsumed by S, for any $i \geq 0$. For the same reason, no s_i is subsumed by S.

Definition 9 Let $\Delta = (W, D)$ be a default theory. For any set S of formulae and any set T of indexed formulae, define $\Gamma^1_\Delta(S,T)$ and $\Gamma^2_\Delta(S,T)$ as the minimal sets satisfying the following conditions:

- $W \subseteq \Gamma^1_\Delta(S,T)$,
- $Th(\Gamma^1_\Delta(S,T)) = \Gamma^1_\Delta(S,T)$,
- $\langle \alpha, \varepsilon \rangle \in \Gamma^2_\Delta(S,T)$ for all $\alpha \in W$,
- if $\delta = [\alpha : \beta_1, \ldots, \beta_n/\gamma] \in D$, $\alpha \in \Gamma^1_\Delta(S,T)$, for all $\varphi \in JUS(\mathcal{D}(T)) \cup \{\beta_1, \ldots, \beta_n\}$, $S \cup \{\gamma\} \nvdash \neg\varphi$ and $\langle \gamma, \delta \rangle$ is not subsumed by T, then
  - $\gamma \in \Gamma^1_\Delta(S,T)$,
  - $\langle \gamma, \delta \rangle \in \Gamma^2_\Delta(S,T)$.

We say that E is a *c-extension* of Δ with respect to F if and only if $E = \Gamma^1_\Delta(E,F)$ and $F = \Gamma^2_\Delta(E,F)$.

As in the case of b-extensions, Γ^1_Δ corresponds, roughly speaking, to beliefs. Γ^2_Δ can be viewed as some sort of a catalog in which all sources of information, that is axioms and applied defaults, are recorded. It essentially contains a trace of the agent's reasoning. Some of the flavor of the idea of recording the "history" of applying defaults is captured by some work in inheritance.[3]

The following theorem gives more intuitive characterization of a c-extension and closely corresponds to the theorems proved in (Reiter 1980) and (Lukaszewicz 1988).

Theorem 1 If $\Delta = (W, D)$ is a default theory, then E is a c-extension for Δ with respect to a set F of indexed formulae if and only if

$$E = \bigcup_{i=0}^{\infty} E_i \quad \text{and} \quad F = \bigcup_{i=0}^{\infty} F_i$$

where

$$E_0 = W \quad \text{and} \quad F_0 = \{\langle \alpha, \varepsilon \rangle : \alpha \in W\},$$

for $i \geq 0$

$$E_{i+1} = Th(E_i) \cup \{\gamma : \boldsymbol{\Sigma}\} \text{ and } F_{i+1} = F_i \cup \{\langle \gamma, \delta \rangle : \boldsymbol{\Sigma}\}$$

and $\boldsymbol{\Sigma}$ stands for the condition that there is $\delta = [\alpha : \beta_1, \ldots, \beta_n/\gamma] \in D$ such that $\alpha \in E_i$, for every $\varphi \in JUS(\mathcal{D}(F)) \cup \{\beta_1, \ldots, \beta_n\}$, $E \cup \{\gamma\} \nvdash \neg\varphi$ and $\langle \gamma, \delta \rangle$ is not subsumed by F.

A simple conclusion that follows from Definition 9 is that every c-extension is a set of formulae that are entailed by axioms and consequents of applied defaults of an underlying default theory.

Theorem 2 Suppose E is a c-extension of some default theory with respect to F. Then $E = Th(\mathcal{F}(F))$.

As we may expect, c-extensions, just as b-extensions, also satisfy the next two theorems.

Theorem 3 Every default theory has a c-extension.

Theorem 4 (Semimonotonicity) Let D and D' be sets of defaults such that $D \subseteq D'$ and suppose that $\Delta = (W, D)$ has a c-extension E. Then $\Delta' = (W, D')$ has a c-extension E' such that $E \subseteq E'$.

There are many other similarities between the notions of a b- and c-extension. It is worth mentioning, for example, that like b-extensions, c-extensions need not to be maximal sets of beliefs, and hence, two different c-extensions do not necessarily have to represent orthogonal sets of beliefs. As an example, let us consider the theory

$$(\emptyset, \{d_1 = [\top : q/\neg p], d_2 = [\top : r/\neg p \wedge \neg q]\}). \qquad (2)$$

This theory has two c-extensions $E_1 \subset E_2$ with respect to F_1 and F_2, respectively, where $E_1 = Th(\{\neg p\})$, $F_1 = \{\langle \neg p, d_1 \rangle\}$, $E_2 = Th(\{\neg p \wedge \neg q\})$ and $F_2 = \{\langle \neg p \wedge \neg q, d_2 \rangle\}$. E_1 and E_2 are also the only b-extensions of (2). The only a-extension of (2) is E_2.

[3]Some review material concerning this issue can be found in (Rychlik 1989).

There are default theories, however, some of whose b-extensions are not maximal sets of beliefs, whereas all their c-extensions are. Let us examine an example from (Łukaszewicz 1988) in which the following theory is presented:

$$(\{p\}, \{d_1 = [\top : r/p], d_2 = [\top : q/\neg r]\}). \qquad (3)$$

This theory has two b-extensions $E_1 \subset E_2$ with respect to F_1 and F_2, respectively, where $E_1 = Th(\{p\})$, $F_1 = \{r\}$, $E_2 = Th(\{p, \neg r\})$, and $F_2 = \{q\}$. But, E_2 with respect to $\{\langle \neg r, d_2 \rangle\}$ is the only c-extension of (3). E_2 is also the only a-extension of (3).

Although examples (2) and (3) are very similar, there is one important difference. In (2), a rationally thinking agent believes that p holds, but she can choose between two arguments that support this belief. One of them blocks the assumption that r holds. In (3), there is no need for her to justify p using defaults, because she already knows it, and therefore she can conclude r. We say that in such a case, the agent shows the ability to *rationally maximize* her own beliefs applying as few defaults as possible. These intuitions are formally described below.

Definition 10 Let E be an extension[4] of a default theory Δ. By $GD(E, \Delta)$ we will understand all defaults used in generating E. If E is a c-extension of Δ with respect to F, then $GD(E, \Delta) = \mathcal{D}(F)$.

Definition 11 Let $\Delta = (W, D)$ be a default theory and E an extension of Δ. We say that a default δ is *superfluous* in $GD(E, \Delta)$ if and only if $\langle con(\delta), \delta \rangle$ is subsumed by $\{\langle \alpha, \varepsilon \rangle : \alpha \in W\} \cup \{\langle con(\xi), \xi \rangle : \xi \in GD(E, \Delta)\}$.

Definition 12 Let E be an extension of a default theory $\Delta = (W, D)$. We say that E is a *rationally maximal set of beliefs* if and only if there is no extension $E' \subset E$ of Δ such that there is a default $\delta \in GD(E', \Delta)$ which is superfluous in $GD(E', \Delta)$.

Theorem 5 Every c-extension is a rationally maximal set of beliefs.

One may wonder whether our approach offers any advantages over Reiter's or Lukaszewicz's simpler formulations. The main advantage of a c-extension over an a-extension is that the former is guaranteed to exist for any default theory, which is not the case for the latter. On the other hand, for many representational problems which involve default reasoning, our new formalization might give solutions that are less intuitive than those supported by Reiter's or Lukaszewicz's formalizations (and vice versa).

In most knowledge representation systems, the intuitive meaning of a default $[\alpha : \beta_1, \ldots, \beta_n/\gamma]$ is that *normally, if α is satisfied, then also γ is satisfied unless some β_i is assumed to be false.* Let us return to the example given above. The first default in (3) says that normally p is satisfied unless r is assumed to be false. In other words, the cases where p is satisfied and r is known to be false are outnumbered by the cases in which p and r are both satisfied. Given (3), a common sense reasoner would conclude that, most probably, r is satisfied. In this case, she would prefer E_1 over E_2. Suppose, however, that she is presented the following default theory:

$$(\ \{p \wedge s\},\ \{\ d_1 = [\top : r/p],\ d_2 = [\top : q/\neg r],\ d_3 = [p \wedge s : \neg r/\neg r]\ \}\ ). \qquad (4)$$

The above theory is similar to (3), but additionally d_3 says that normally r is false whenever p and s are satisfied. Since the only axiom of (4) says that p and s are true, r is very likely false. A common sense reasoner would, therefore, choose an extension which supports $\neg r$. Notice that (4) has two b-extensions $E_1 = Th(\{p \wedge s\})$, which does not conform to our intuitions, and $E_2 = Th(\{p \wedge s, \neg r\})$. E_2 is the only a-extension and c-extension of (4).

Finally, let us consider an example of a default theory

$$(\ \{e \wedge ce\},\ \{\ d_1 = [e : g \wedge \neg re/g],\ d_2 = [ce : re \wedge \neg r/re]\ \}\ ) \qquad (5)$$

given in (Besnard 1989). Assume the following interpretation of the predicate symbols:

- e — Clyde is an elephant,
- ce — Clyde is a circus elephant,
- re — Clyde is a royal elephant,
- g — Clyde is gray,
- r — Clyde is rare.

This theory has two b-extensions which are also the only c-extensions. One of them contains the fact that Clyde is a royal elephant. The other one supports the assumption that Clyde is gray, which seems to be unintuitive. The only a-extension of (5) contains the fact that Clyde is a royal elephant.

The above examples suggest that neither Reiter's, nor Lukaszewicz's, nor our approach to default reasoning can be preferred overall.

Let us end our discussion with the following result:

Theorem 6 If A, B and C are sets of a-extensions, b-extensions and c-extensions of some default theory, respectively, then $A \subseteq C \subseteq B$.

Semantics

In this section, we will provide a model-theoretic semantics of c-extensions for default theories. We will use a technique proposed in (Etherington 1986) and (Lukaszewicz 1988).

Any nonmonotonic logic can be viewed as a result of transforming some base standard logic by a selection strategy defined on models.[5] For default logics,

[4] We mean here either an a-, b-, or c-extension.

[5] See, for example, (Rychlik 1990).

this selection strategy consists, roughly speaking, in restricting the set of models of the underlying default theory. Suppose $\Delta = (W, D)$ is a default theory. Applying any default $\delta_1 \in D$ whose conclusion does not follow directly from W causes the class $\mathfrak{M}_{\{\}}$ of all models of W to be narrowed to the class $\mathfrak{M}_{\{\delta_1\}}$ of models that satisfy the consequent of δ_1. If we further apply another default $\delta_2 \in D$, the class $\mathfrak{M}_{\{\delta_1\}}$ will be narrowed to the class $\mathfrak{M}_{\{\delta_1,\delta_2\}}$ which contains those models from $\mathfrak{M}_{\{\delta_1\}}$ that additionally satisfy the consequent of δ_2. And so on. In this way we will obtain a sequence $\mathfrak{M}_{\{\}} \subseteq \mathfrak{M}_{\{\delta_1\}} \subseteq \mathfrak{M}_{\{\delta_1,\delta_2\}} \subseteq \ldots$ Every extension of Δ has its models among the maximal elements of such sequences. Let us formalize these intuitions as follows.

Definition 13 Let $\mathfrak{M}$ be a set of models for some set of formulae and F a set of indexed formulae. A default $\delta = [\alpha : \beta_1, \ldots, \beta_n/\gamma]$ is *applicable* with respect to $(\mathfrak{M}, F)$ if and only if for every $\mathfrak{m} \in \mathfrak{M}$, $\mathfrak{m} \models \alpha$, for every $\varphi \in JUS(\mathcal{D}(F)) \cup \{\beta_1, \ldots, \beta_n\}$ there is $\mathfrak{m} \in \mathfrak{M}$ such that $\mathfrak{m} \models \gamma \wedge \varphi$ and $\langle \gamma, \delta \rangle$ is not subsumed by F.

Definition 14 Let $\mathfrak{M}$ and F be as above. For a default $\delta = [\alpha : \beta_1, \ldots, \beta_n/\gamma]$, we construct two sequences $X_0, X_1, \ldots$ and $Y_0, Y_1, \ldots$ as follows:

$$X_0 = \{F \cup \{\langle \gamma, \delta \rangle\}\},$$
$$Y_0 = \{\varsigma \in X_0 \setminus \mathcal{K}(F) : \varsigma \text{ is subsumed by } X_0\}$$

and for $i \geq 0$

$$X_{i+1} = \bigcup_{x \in X_i} \Big(\bigcup_{\varsigma \in Y_i} \{x \setminus \{\varsigma\}\} \Big),$$
$$Y_{i+1} = \{\varsigma \in X_{i+1} \setminus \mathcal{K}(F) : \varsigma \text{ is subsumed by } X_{i+1}\}.$$

Put

$$X = \bigcup_{i=0}^{\infty} X_i.$$

The *result* of δ in $(\mathfrak{M}, F)$ (written $\delta(\mathfrak{M}, F)$) is either:

(a) $\{(\mathfrak{M} \setminus \{\mathfrak{m} : \mathfrak{m} \models \neg\gamma\}, x) : x \in X\}$ if and only if δ is applicable with respect to $(\mathfrak{M}, F)$ and $\langle \gamma, \delta \rangle \notin F$,

(b) $\{(\mathfrak{M}, F)\}$ otherwise.

Definition 15 Let P be a set of pairs $(\mathfrak{M}, F)$ where $\mathfrak{M}$ and F are defined as above. We say that

$$\delta(P) = \bigcup_{p \in P} \delta(p)$$

is a *result* of δ in P.

Definition 16 Let $\mathfrak{M}$ and F be as above. We say that $(\mathfrak{M}, F)$ is *stable* with respect to a set D of defaults if and only if for all $\delta \in D$, $\delta(\mathfrak{M}, F) = \{(\mathfrak{M}, F)\}$ and every element of $F \setminus \mathcal{K}(F)$ is not subsumed by F.

Definition 17 Let $\mathfrak{M}$ and F be as above. Suppose $\langle \delta_i \rangle$ is a sequence of defaults. By $\langle \delta_i \rangle(\mathfrak{M}, F)$ we denote $\bigcup R_i$ where $R_0 = \{(\mathfrak{M}, F)\}$ and for $i \geq 0$, $R_{i+1} = \delta_i(R_i)$.

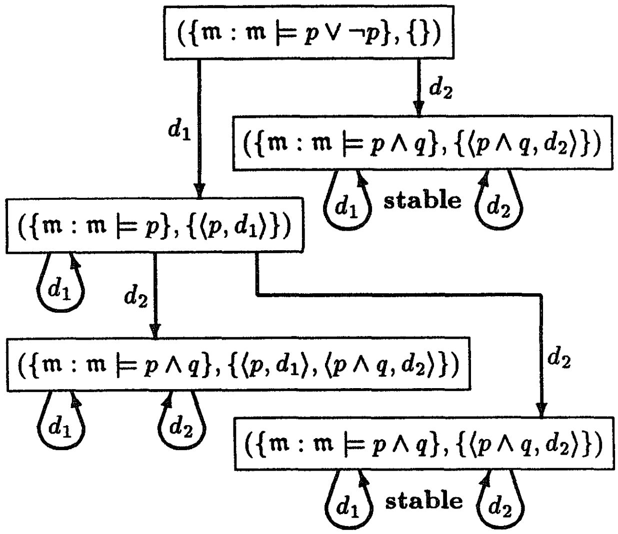

Figure 1: Network corresponding to (6).

Definition 18 Let $\mathfrak{M}$ and F be as above. Suppose $\mathfrak{N}$ is a set of models for some set of formulae and D a set of defaults. We say that $(\mathfrak{M}, F)$ is *accessible* from $\mathfrak{N}$ with respect to D if and only if there is a sequence $\langle \delta_i \rangle$ of defaults from D such that $(\mathfrak{M}, F) \in \langle \delta_i \rangle(\mathfrak{N}, \mathcal{K}(F))$.

Theorem 7 (Soundness) If E is a c-extension for a default theory $\Delta = (W, D)$, then there is some set F of indexed formulae such that $(\{\mathfrak{m} : \{\mathfrak{m} \models E\}, F)$ is stable with respect to D and accessible from the set of models of W.

Theorem 8 (Completeness) Let $\Delta = (W, D)$ be a default theory. If $(\mathfrak{M}, F)$ is stable with respect to D and accessible from the set of models of W, then $\mathfrak{M}$ is the set of models for some c-extension of Δ.

We can envisage the semantics of default theories as transition networks (Etherington 1986, Lukaszewicz 1988) whose nodes stand for pairs $(\mathfrak{M}, F)$ and arcs are labeled by defaults. If $\Delta = (W, D)$ is a default theory then $\mathfrak{M}$ represents some subset of all models of W. The root node of the network for Δ is the node $(\{\mathfrak{m} : \mathfrak{m} \models W\}, \{\langle \alpha, \varepsilon \rangle : \alpha \in W\})$. From the node $(\mathfrak{M}, F)$, for every $\delta = [\alpha : \beta_1, \ldots, \beta_n/\gamma] \in D$, arcs labeled δ lead to the nodes $\delta(\mathfrak{M}, F)$ as it is defined in Definition 14.

Figure 1 represents a network corresponding to a default theory

$$(\emptyset, \{d_1 = [\top : r/p], d_2 = [\top : s/p \wedge q]\}). \qquad (6)$$

On Figure 2 a network corresponding to (3) is shown.

Conclusion

In this paper we introduced an alternative formalization of default reasoning. We presented, in the form of

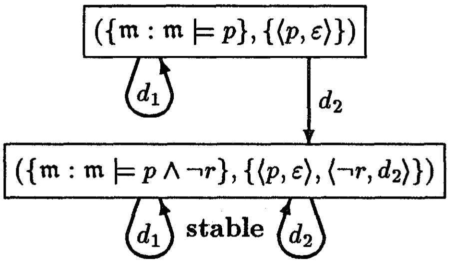

Figure 2: Network corresponding to (3).

a fixed-point construct, a new definition of the notion of an extension for default theories. Then we characterized this notion from the semantic point of view. We also made an observation that extensions redefined in such way represent so-called rationally maximal sets of beliefs. We motivated our approach by noticing that perfect reasoners tend not to use redundant arguments while explaining their beliefs about the world. Of course, we know that there might be no single formalism in which we could reflect our intuitions about how a perfectly reasoning agent should draw her inferences. A clash of different intuitions in formalizing multiple inheritance with exceptions is a good example supporting this observation. It seems, however, that it is not difficult to modify a default logic so that it can properly handle many of these intuitions.

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