A Temporal Terminological Logic*

Albrecht Schmiedel
Technische Universität Berlin
Sekr. FR 5-12, Projekt KIT-BACK
Franklinstraße 28/29
D-1000 Berlin 10, West-Germany
schmied@db0tuil1.bitnet

Abstract

An attempt is made to integrate three well-known formalisms of knowledge representation: terminological logic in the tradition of KL-ONE, the temporal logic of Shoham, and Allen's interval calculus. Drawing on each of these sources, a temporal terminological logic is proposed which combines structural with temporal abstraction. A formal semantics is provided, and some hints are given for exploring the computational properties of reasoning in the formalism.

Introduction

Terminological logics in the tradition of KL-ONE [Brachman & Schmolze 1985] as well as temporal reasoning have both received considerable attention within the knowledge representation community in the last years. However, there has never been a serious attempt to integrate these two fields. Although in a number of projects using terminological logic (e.g. [Poesio 1988]) the problem of representing time has arisen, notably in the context of tense in natural language understanding, the approaches taken there and the partial solutions found have not culminated in a syntactically and semantically well-defined temporal variant of a terminological logic. The theoretical framework described in the following is the foundation of a (future) temporal extension of BACK, a knowledge representation system based on terminological logic being developed in our project [Peltason et al. 89].

The approach for integrating time into a terminological formalism which I am proposing here draws on three ingredients. First of all on terminological logic itself, the appealing features of which are completely preserved in the temporal variant. The temporal capabilities come straightforwardly by adding some new term-forming constructs. The model-theoretic semantics are accordingly amended, and remain unchanged for non-temporal terms. Compositionality is unaffected; there are no restrictions on the combination of temporal and non-temporal terms.

The second ingredient is concerned with the elementary combination of temporal and non-temporal objects. Following [Shoham 1987] I will keep temporal and non-temporal components of the language separate, giving time a special status in the formalism. The role of 'TRUE' in his logic is played by a new term-forming construct '(**at** *interval concept*)' which denotes the set of all individuals that are in the denotation of *concept* at the time *interval*. Denotations of concepts are interpreted at intervals, not at points. Also as in Shoham's approach there is no commitment with respect to the *property/event/process trichotomy* in the basic framework. The denotation of a concept at one interval is essentially unrelated to its denotation at other intervals.

Thirdly, for expressing temporal relationships and constraints, I rely on Allen's interval calculus [Allen 83] extended by some additional constraint types for dealing with durations, absolute times, and the granularity of intervals. This restricts the range of expressible temporal constraints compared with a full-fledged temporal logic, but for this subset specialized algorithms are available making an efficient treatment at least for a broad range of 'non-puzzle-mode' cases conceivable, which is a prerequisite for a knowledge representation (KR) service.

Syntax and Introductory Examples

Figure 1 shows the syntax of some of the basic concept- and role-forming constructs common to most terminological logics of the KL-ONE family: **and**, **all**, **atleast**, **atmost** for concepts, and **and**, **domain**, **range** for roles. Note that the (**restrict** *role concept*) construct found in some systems is equivalent to (**and** *role* (**range** *concept*)) in our syntax. The new term-forming operators involving time are **at**, **sometime**, and **alltime**.[1] The syntax for time intervals and time

*This work was supported by the Commission of the European Communities as part of ESPRIT Project 311.

[1] I have deliberately included a fairly expressive set of role-forming constructs to demonstrate the expressive potential of the formalism, disregarding for the time being various possible trade-offs between expressivity and the

concept ::= *atomic-concept*
| (**and** *concept*$^+$)
| (**all** *role concept*)
| (**atleast** *min role*)
| (**atmost** *max role*)
| (**at** *interval concept*)
| (**sometime** (*interval-variable*$^+$) *time-net concept*)
| (**alltime** (*interval-variable*$^+$) *time-net concept*)

role ::= *atomic-role*
| (**and** *role*$^+$)
| (**domain** *concept*)
| (**range** *concept*)
| (**at** *interval role*)
| (**sometime** (*interval-variable*$^+$) *time-net role*)
| (**alltime** (*interval-variable*$^+$) *time-net role*)

Figure 1: Syntax for Concepts and Roles with Temporal Structure

nets is found in Figure 2.

In standard terminological logics using the non-temporal constructs of the syntax concepts can be formed such as

(**and** man
(**atleast** 1 (**and** child (**range** female)))
(**all** child grown-up)),

with the intuitive meaning *a man with atleast one child, which is a female, whose children are all grown-up.* According to the model-theoretic semantics for the term-forming constructs, the extension of a term is strictly a function of the extensions of its subterms and of the extensions of the atomic concepts or roles it contains, in this example the concepts man, female, grown-up, and the role child.

An important feature that all KR systems based on terminological logics provide is the possibility of introducing new atomic concepts and roles by definition, for example

daughter := (**and** child (**range** female)).

After this definition, the new name can be used in other terms and definitions as an abbreviation for the defining term.

Before looking at some examples using the new constructs involving time, let us consider the meaning of non-temporal terms. For example, if we define

car-owner := (**atleast** 1 (**and** own (**range** car))),

what is the meaning of car-owner within the temporal framework? The answer is that all terms have to be evaluated with respect to a particular interval of time. Consequently, car-owner does no more denote a set of individuals fulfilling the definition, but in fact a function that assigns such a set of individuals to every interval. Using the **at**-construct, we can *bind* the time of evaluation of a term to a specific interval. For example,

(**at** 'August 1990' car-owner)

denotes the set of car-owners at the interval denoted by the interval constant 'August 1990'.

Every term can be rewritten as (**at** *NOW term*). *NOW* is the special reference interval which represents the index at which the term is evaluated. Thus, the index implicit in every non-temporal term can be made explicit. Every **at**-term creates an evaluation environment whereby every (explicit and implicit) occurrence of *NOW* in the embedded term is bound to the interval specified in the **at**-term. Taking the last example, if we expand the definition of car-owner and bind the implicit *NOW*'s, we get

(**atleast** 1 (**and** (**at** 'August 1990' own)
(**range** (**at** 'August 1990' car)))).

Note that the outer **at**-terms are redundant (and have therefore been deleted in the example), since there are no more embedded *NOW*'s that the time interval could be bound to.

We can identify two classes of terms: *time-indexed terms* whose denotation depends on a particular value of *NOW* and *universal terms* whose denotation is the same for all values of *NOW*. Obviously, all terms that contain no unbound (implicit or explicit) occurrences of *NOW* are necessarily universal, and as a consequence an **at**-term embedding a universal term is redundant and can be replaced by the term itself.

With the **at**-construct and temporal constants alone, expressivity is very restricted. The potential for temporal abstraction comes with the ability to express abstract temporal patterns, which classify objects in terms of their pattern of change. In order to achieve

computational complexity of classification algorithms. I could also have included negation and disjunction (as some terminological logics do), but that contributes nothing to the problem at hand of integrating time.

time-net	::=	*time-constraint*
	|	(**and** *time-constraint*$^+$)
time-constraint	::=	(*interval-relation interval interval*)
	|	(*comparison interval duration-constant*)
	|	(*granularity interval*)
interval-relation	::=	**equal** | **before** | **after** | **meets** | **met-by** | **overlaps** | **overlapped-by** | **starts** | **started-by** | **finishes** | **finished-by** | **during** | **contains** | (**or** *interval-relation*$^+$)
comparison	::=	$<$ | $\leq$ | $=$ | $\geq$ | $>$
granularity	::=	**sec** | **min** | **hour** | **day** | **month** | **year** | ...
interval	::=	*interval-variable* | *interval-constant* | *NOW*
interval-variable	::=	x | y | z | ... | x_1 | x_2 | x_3 | ...
interval-constant	::=	'1987' | '3/12/1987 14:04:12' | ...
duration-constant	::=	'5d 2h 4min 33sec' | ...

Figure 2: Syntax for Time Nets

this, temporal variables and means of expressing constraints over these variables are necessary.[2] Temporal variables are introduced by the temporal quantifiers **sometime** and **alltime** together with a set of constraints, a *time net*, over these variables. Three kinds of constraints are allowed by the syntax for time nets according to Figure 2: relations between pairs of intervals using Allen's basic interval relations and disjunctions of these, metric constraints on single intervals, and granularity constraints requiring an interval to take values that are multiples of some time unit. Absolute bounds can be imposed on an interval by using interval constants in interval relations.

The following term denotes the set of individuals that were car-owners at an interval sometime before *NOW*:

```
former-car-owner := (sometime (x)
                       (before x NOW)
                       (at x car-owner)).
```

former-car-owner can be equivalently expressed using a temporal role:

```
former-car-owner := (atleast 1 have-owned-a-car),

have-owned-a-car := (sometime (x)
                       (before x NOW)
                       (at x
                          (and own (range car))))).
```

Two individuals are related by have-owned-a-car if they were related by own at some interval before *NOW* and if at that time the second was an instance of car.

The following term correctly applies to an individual *NOW* if at a point during *NOW* he ceases to be a car-owner and becomes a bike-owner:

```
(and (sometime (x y)
        (and (start x NOW)
             (finishes NOW y)
             (meets x y))
        (and (at x car-owner)
             (at y bike-owner)))
     (alltime (x)
        (during x NOW)
        (at x (atmost 1
                 (and own (range vehicle)))))).
```

It is important to realize that in the **sometime**-term there is nothing that expresses 'ceases to be' or 'becomes'; being car-owner and bike-owner at the same time throughout *NOW* would be perfectly consistent. It is only in conjunction with the **alltime** term, which restricts the number of role-fillers for the own-a-vehicle role to atmost one for all times during *NOW*, and with the assumption that car and bike are subsumed by vehicle, that this interpretation is ruled out.

In the previous examples, time nets only used interval relations to constrain temporal variables. The following time net additionally uses metric constraints on the duration of intervals, and granularity predicates:

```
(and (day x) (= x '24h')
     (day y) (= y '24h')
     (meets x y)
     ((or starts finishes during equal) x NOW)
     ((or starts finishes during equal) y NOW))
```

It constrains x and y to be consecutive days within *NOW*. The granularity constraint (**day** x) above restricts x to take only values that are started and fin-

[2] The introduction of variables in terms seems counter to the spirit of KL-ONE-like languages; but the only variable-free temporal language I could think of is equivalent to the subset of the one presented here where there is only one variable per **sometime**- or **alltime**-term. That appears overly restrictive; not even a covering of *NOW* by a sequence of meeting intervals can be expressed.

$$
\begin{array}{rcl}
\mathcal{E}[(\mathbf{and}\ c_1 \ldots c_n)]^t_{\mathcal{I}} & = & \bigcap_{i=1}^{n} \mathcal{E}[c_i]^t_{\mathcal{I}} \\
\mathcal{E}[(\mathbf{all}\ r\ c)]^t_{\mathcal{I}} & = & \{d \in \mathcal{D} : \mathcal{E}[r]^t_{\mathcal{I}}(d) \subseteq \mathcal{E}[c]^t_{\mathcal{I}}\} \\
\mathcal{E}[(\mathbf{atleast}\ m\ r)]^t_{\mathcal{I}} & = & \{d \in \mathcal{D} : |\mathcal{E}[r]^t_{\mathcal{I}}(d)| \geq m\} \\
\mathcal{E}[(\mathbf{atmost}\ m\ r)]^t_{\mathcal{I}} & = & \{d \in \mathcal{D} : |\mathcal{E}[r]^t_{\mathcal{I}}(d)| \leq m\} \\
\mathcal{E}[(\mathbf{at}\ x\ c)]^t_{\mathcal{I}} & = & \begin{cases} \mathcal{E}[c]^{\mathcal{I}(x)}_{\mathcal{I}} & \text{if } x \text{ is a variable} \\ \mathcal{E}[c]^t_{\mathcal{I}} & \text{if } x = NOW \\ \mathcal{E}[c]^{\mathcal{M}(x)}_{\mathcal{I}} & \text{if } x \text{ is a constant} \end{cases} \\
\mathcal{E}[(\mathbf{sometime}\ X\ TC\ c)]^t & = & \{d \in \mathcal{D} : \exists \mathcal{I} \in \mathcal{I}^*(\langle TC, X \cup \{NOW\}\rangle)_{NOW \mapsto t} \quad d \in \mathcal{E}[c]^t_{\mathcal{I}}\} \\
\mathcal{E}[(\mathbf{alltime}\ X\ TC\ c)]^t & = & \{d \in \mathcal{D} : \forall \mathcal{I} \in \mathcal{I}^*(\langle TC, X \cup \{NOW\}\rangle)_{NOW \mapsto t} \quad d \in \mathcal{E}[c]^t_{\mathcal{I}}\} \\
\mathcal{E}[(\mathbf{domain}\ c)]^t_{\mathcal{I}} & = & \mathcal{E}[c]^t_{\mathcal{I}} \times \mathcal{D} \\
\mathcal{E}[(\mathbf{range}\ c)]^t_{\mathcal{I}} & = & \mathcal{D} \times \mathcal{E}[c]^t_{\mathcal{I}}
\end{array}
$$

Figure 3: Semantic Conditions for Extension Functions

ished by a day. Without this constraint, x could be any interval with a duration of 24 hours, due to the metric constraint (= x '24h'), but not necessarily coinciding with a full day of the calendar. On the other hand, leaving away the duration constraint, x could be any interval starting and ending with a full day.

Semantics

In terminological logics in the tradition of KL-ONE it has become customary to provide a model-theoretic account of the semantics (e.g. [Schmolze 1989, Nebel 1989]). I will follow this tradition. Before spelling out the semantics for concepts and roles, the semantics associated with temporal constraints must be clarified. For the present purpose, I will assume a discrete time model and interpret all time intervals as pairs of integers, and define the domain of time intervals as follows: $\mathcal{T} \stackrel{\text{def}}{=} \{\langle i_1, i_2\rangle \mid i_1 < i_2, i_1, i_2 \in \mathbf{Integer}\}$. Thus, consecutive integers form the smallest, non-decomposable intervals, the *moments* in the sense of [Allen & Hayes 1985]. For time constraints according to Figure 2, I will assume a *fixed model* $\mathcal{M}$ which maps interval constants to elements of $\mathcal{T}$, duration constants to subsets of $\mathcal{T}$, comparison operators and interval relations to sets of pairs of elements of $\mathcal{T}$, and granularity predicates to subsets of $\mathcal{T}$, such that the intuitive meaning of these constructs is adequately mirrored (for example, that $\langle \mathcal{M}[\text{'August 1990'}], \mathcal{M}[\text{'September 1990'}]\rangle \in \mathcal{M}[\mathbf{meets}]$, $\mathcal{M}[\text{'3/12/1990'}] \in \mathcal{M}[\mathbf{day}]$, etc.).

An *interpretation*[3] of a time net $\langle TC, X\rangle$, where TC is a set of constraints and X a set of variables, is a function $\mathcal{I} : X \rightarrow \mathcal{T}$ which satisfies TC (for example, if $(\mathbf{meets}\ x\ y) \in TC$, then $\langle \mathcal{I}(x), \mathcal{I}(y)\rangle \in \mathcal{M}[\mathbf{meets}]$). The ***set of all interpretations*** of the time net $\langle TC, X\rangle$ is denoted by $\mathcal{I}^*(\langle TC, X\rangle)$. The ***set of all interpretations of a time net in all of which x is mapped to the same value*** is denoted by $\mathcal{I}^*(\langle TC, X\rangle)_{x \mapsto t}$.

A *model* for a set of terms with temporal structure defined by the syntax in Figures 1 and 2 is a triple $\langle \mathcal{D}, \mathcal{T}, \mathcal{E}\rangle$ where $\mathcal{D}$ is a set of individuals, $\mathcal{T}$ is the set of time intervals, and $\mathcal{E}$ is a function

$$
\mathcal{E} : \begin{cases} C & \rightarrow & (\mathcal{T} \rightarrow 2^{\mathcal{D}}) \\ R & \rightarrow & (\mathcal{T} \rightarrow 2^{\mathcal{D} \times \mathcal{D}}) \end{cases}
$$

where C are the concept terms and R are the role terms without free variables and after all definitions have been expanded. Thus, each concept (each role) is mapped to a function that assigns sets of individuals (sets of pairs of individuals) to each time interval. For $\langle \mathcal{D}, \mathcal{T}, \mathcal{E}\rangle$ to be a model, the conditions in Figure 3[4] that define the meaning of the syntactic constructs must be met for all $t \in \mathcal{T}$.

Taking into account the extension at each time interval, subsumption can now be defined in the usual way: *For all concepts and roles, c_1 subsumes c_2 iff for all extension functions $\mathcal{E}$ (models) and all time intervals $t \in \mathcal{T}, \mathcal{E}[c_2]^t \subseteq \mathcal{E}[c_1]^t$.*

Kinds of Time Dependency

As already mentioned in the introduction, in the basic semantic framework as spelt out in the last section there are no built-in restrictions on the extensions of primitive concepts or roles. For example, if a pair of individuals are related by the primitive role own at one interval, they may or may not be related by that role in any subinterval. As a consequence, according to the semantics so far,

(**at** 'June 1990' own)

does not subsume

(**at** '1990' own),

and

[3] The notation for time nets and their interpretations owes much to [Nökel 1988].

[4] Notation: $r(d) \stackrel{\text{def}}{=} \{d' \in \mathcal{D} : \langle d, d'\rangle \in r\}$, and $\mathcal{E}[c]^t_{\mathcal{I}} \stackrel{\text{def}}{=} \mathcal{E}(c)(t)$ where all free variables in c are evaluated using $\mathcal{I}$. Note that $\mathcal{I}$ is only needed to evaluate at-terms nested within **sometime**- and **alltime**-terms.

The conditions for and, at, sometime, and alltime for roles have been omitted since they are completely analogous to those for concepts.

(**sometime** (x)
(> x '35 days')
(**at** x own))

does not subsume

(**and** (**at** 'June 1990' own)
(**at** 'July 1990' own)

although intuitively if an own relation holds in an interval it should also be valid in all subintervals, and if it holds over two consecutive intervals it should hold over their union (which, in the second example, is then an interval longer than 35 days). Extra restrictions on extension functions are needed in order to legitimize the subsumptions above. Using the terminology in [Shoham 1987], the possible extensions of the primitive role own should be restricted to being *downward-hereditary*: for all t, $t' \in \mathcal{T}$, t' subinterval of $t, \mathcal{E}[\text{own}]^t \subseteq \mathcal{E}[\text{own}]^{t'}$, and *concatenable*: for all $t, t', t'' \in \mathcal{T}$, t meets t', t starts t'', t' finishes t'', $\mathcal{E}[\text{own}]^t \cap \mathcal{E}[\text{own}]^{t'} \subseteq \mathcal{E}[\text{own}]^{t''}$.

Of course, for other kinds of concepts and roles these restrictions are quite inadequate. For example, for a role average-temperature that relates an object to its average temperature in a certain time interval, the first subsumption relation above should not hold since average temperatures are not inherited to subintervals; the second may be correct though, because if an average remains identical for two consecutive intervals also applies for their union. The latter is certainly not true for other aggregate values which depend on time, such as e.g. sales of a department per month, or annual salary of an employee. For these, another kind of restriction might be more appropiate: a role sales-per-month, for example, should take values only for intervals that correspond to full months, and be empty for all other intervals.

How can these different kinds of restrictions be integrated into our framework? The situation is quite similar to the problem of integrating disjointness restrictions into terminological logics. Should the disjointness of e.g. male and female be treated as definitional and hence be used for validating certain subsumptions, or rather as assertional? On pragmatic grounds, the former alternative is generally chosen. Terminological systems allow disjointness restrictions for primitive concepts and use these for computing subsumption. The same approach could be adopted for restrictions on temporal extensions of primitive concepts or roles. So, in the example above, after declaring the primitive role own as downward-hereditary and concatenable, the subsumptions would be valid.

Computing Subsumption

Subsumption is the central semantic notion in terminological logics; designing sound (and possibly complete) algorithms that compute subsumption is the central issue for providing practical KR services. Until now, no algorithms are available for the temporal variant presented here, but atleast some preliminary hints as to what is involved can be given. Assume CT' is of the form

$$(\textbf{sometime}\ X'\ TC'\ (\textbf{and}\ \ldots(\textbf{at}\ x'_j\ c'_j)\ldots)),$$
$$j \in J, x_j \in X', c'_j \in C'$$

and CT is of the form

$$(\textbf{sometime}\ X\ TC\ (\textbf{and}\ \ldots(\textbf{at}\ x_i\ c_i)\ldots)),$$
$$i \in I, x_i \in X, c_i \in C.$$

Under what conditions does CT' subsume[5] CT? Intuitively, CT' is a more general concept than CT, if its temporal variables are less constrained than those of CT, and for each of its temporal variables there is a corresponding variable in CT such that the associated concept of that variable subsumes the associated concept in CT. Obviously, CT can have additional temporal variables and associated concepts, which specialize it further.

To formalize this notion, let $S : J \rightarrow I$ be a function from indices of CT' to indices of CT. $S_X : X' \rightarrow X$ is defined as $S_X(x'_j) = x_{S(j)}$, and $S_C : C' \rightarrow C$ is defined as $S_C(c'_j) = c_{S(j)}$. S must always map NOW to NOW, i.e. $S_X(NOW) = NOW$.

The notion of 'less constrained temporal variables' is captured by the following definition: A time net $\langle TC', X'\rangle$ ***subsumes*** another time net $\langle TC, X\rangle$ wrt a variable mapping $S_X : X' \rightarrow X$ iff for all $\mathcal{I} \in \mathcal{I}^*(\langle TC, X\rangle)$ there exists an $\mathcal{I}' \in \mathcal{I}^*(\langle TC', X'\rangle)$ such that for all $x \in X'$, $\mathcal{I}'(x) = \mathcal{I}(S_X(x))$.

We can now express the condition above more formally: CT' subsumes CT iff there exists a mapping $S : J \rightarrow I$ such that $\langle TC', X'\rangle$ subsumes $\langle TC, X\rangle$ wrt S_X, and c'_j subsumes $S_C(c'_j)$. The subsumption of time nets guarantees that for every interpretation of TC there is a corresponding interpretation of TC', and the second condition states that for each interval of that interpretation the corresponding concepts are in the subsumption relation. Thus, all instances of CT must also be instances of CT'.

Assuming for the moment that the c_j and c_i are all non-temporal concepts and we can therefore apply known subsumption algorithms, we are left with the problem of determining subsumption between two time nets. Here we can utilize known algorithms for constraint propagation in temporal networks (e.g. [Schmiedel 1988, Valdéz-Pérez 1987]). These algorithms compute the most specific constraints derivable from the given ones, and at the same time check whether the network is consistent. Subsumption can easily be checked—modulo incompleteness of constraint propagation—relative to a given variable mapping. All constraints in the subsuming network must have corresponding ones—via the mapping—in the subsumed network which are at least as restrictive.

[5] Additional restrictions as mentioned in the last section are not taken into account in the following.

Conclusion

Of course complete and tractable subsumption algorithms for the whole language and for the standard semantics presented here cannot be expected. In Allen's interval calculus on its own, which is a subset of our temporal constraint language, determining all consequences of a set of constraints is NP-hard [Vilain & Kautz 1986]. And even for moderately expressive languages the non-temporal part is intractable [Nebel 1988]. That does not render these formalisms useless. On the one hand, it remains to be seen to what extent normal cases in practical applications can be handled even by complete algorithms. On the other hand, algorithms for computing subsumption in terminological logics that are incomplete with respect to standard semantics are increasingly being characterized as complete with respect to a weakened semantics [Schild 1989, Patel-Schneider 1988]; approximative algorithms are also studied in the field of temporal reasoning [van Beek 1989]. These developments are a reasonable starting point for developing subsumption algorithms for temporal terminological logics.

References

[Allen 83] James F. Allen, Maintaining Knowledge About Temporal Intervals, *Communications of the ACM* 26(11), 832–843, 1983.

[Allen & Hayes 1985] James F. Allen, Patrick J. Hayes, A Common-Sense Theory of Time, *Proc. 9th IJCAI*, Los Angeles (Cal.), 528–531, 1985.

[van Beek 1989] Peter van Beek, Approximation Algorithms for Temporal Reasoning, *Proc. 11th IJ-CAI*, Detroit (Mich.), 1291–1296, 1989.

[Brachman & Schmolze 1985] Ronald J. Brachman and James G. Schmolze, An overview of the KL-ONE knowledge representation system, *Cognitive Science* 9(2), 171–216, 1985.

[Nebel 1988] Bernhard Nebel, Computational Complexity of Terminological Reasoning in BACK, *Artificial Intelligence* 34, 371–383, 1988.

[Nebel 1989] Bernhard Nebel, Reasoning and Revision in Hybrid Representation Systems, Ph. D. Thesis, Universität des Saarlandes, Saarbrücken, 1989. To appear in Lecture Notes in Artificial Intelligence, Springer, Berlin.

[Nökel 1988] Klaus Nökel, Convex Relations Between Time Intervals, SEKI Report SR-88-17, Universität Kaiserslautern, 1988.

[Patel-Schneider 1988] Peter F. Patel-Schneider, Adding Number Restrictions to a Four-Valued Terminological Logic, *Proc. AAAI-88*, Saint Paul (Minn.), 485–490, 1988.

[Peltason et al. 89]
Christof Peltason, Albrecht Schmiedel, Carsten Kindermann, and Joachim Quantz, The BACK System Revisited, KIT Report 75, Fachbereich Informatik, Technische Universität Berlin, 1989.

[Poesio 1988] Massimo Poesio, Towards a Hybrid Representation of Time, *Proc. ECAI-88*, Munich, 1988.

[Schild 1989] Klaus Schild, Towards a Theory of Frames and Rules, KIT Report 76, Fachbereich Informatik, Technische Universität Berlin, 1989.

[Schmolze 1989] James G. Schmolze, The Language and Semantics of NIKL, Technical Report 89-4,Tufts University, Medford (Mass.), September 1989.

[Schmiedel 1988] Albrecht Schmiedel, Temporal Constraint Networks, KIT Report 69, Fachbereich Informatik, Technische Universität Berlin, 1988.

[Shoham 1987] Yoav Shoham, Temporal Logics in AI: Semantical and Ontological Considerations, *Artificial Intelligence* 33(1), 89–104, 1987.

[Vilain & Kautz 1986] Marc B. Vilain, Henry Kautz, Constraint Propagation Algorithms for Temporal Reasoning, *Proc. AAAI-86*, Philadelphia (Penn.), 377–382, 1986

[Valdéz-Pérez 1987] Raúl E. Valdéz-Pérez 87, The Satisfiability of Temporal Constraint Networks, *Proc. AAAI-87*, Seattle (Wash.), 256–260, 1987.