

CLOSED FORM SOLUTION TO THE STRUCTURE FROM MOTION PROBLEM FROM LINE CORRESPONDENCES

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ABSTRACT

A theory is presented for the computation of three dimensional motion and structure from dynamic imagery, using only line correspondences. The traditional approach of corresponding microfeatures (interesting points—highlights, corners, high curvature points, etc.) is reviewed and its shortcomings are discussed. Then, a theory is presented that describes a closed form solution to the motion and structure determination problem from line correspondences in three views. The theory is compared with previous ones that are based on nonlinear equations and iterative methods.

1. Introduction

The importance of the estimation of the three dimensional motion of a moving object (or of the sensor) from a sequence of images in robotics (visual input to a manipulator, proprioceptive abilities, navigation, structure computation for recognition, etc.) can hardly be overemphasized.

Up to now there have been three approaches toward the solution of the problem of computation of three dimensional motion from a sequence of images:

- 1) The first method assumes the dynamic image to be a three dimensional function of two spatial arguments and a temporal argument. Then, if this function is locally well behaved and its spatiotemporal gradients are computable, the image velocity or optical flow may be computed [7][9] [31].
- 2) The second method considers cases where the motion is "large" and the previous technique is not applicable. In these instances the measurement technique relies upon isolating and tracking features in the image through time. These features can be microfeatures (highlights, corners, points of high curvature, interest points) or macrofeatures (contours, areas, lines, etc.). In other words, operators are applied to both images which output a set of features in each image, and then the correspondence problem between these two sets of features has to be solved (i.e. finding which features in both dynamic images are projections of the same world feature).

In both of the above approaches, after the optic flow field, the discrete displacement field (which can be sparse), or the correspondence between macrofeatures is

computed, algorithms are constructed for the determination of the three dimensional motion, based on the image flow or on the correspondence [29] [1] [5] [6] [8] [13] [3] [17] [23] [24] [27] [22] [19] [14] [18] [2].

- 3) In the third method, the three dimensional motion parameters are directly computed from the spatial and temporal derivatives of the image intensity function. In other words, if f is the intensity function and (u,v) the optic flow at a point, then the equation $f_x u + f_y v + f_t = 0$ holds approximately. All methods in this category are based on substitution of the optic flow values in terms of the three dimensional motion parameters in the above equation, and there is promising work in this direction. [10] [20] [4] Also, there is work on "correspondenceless" motion detection in the discrete case, where a set of points is put into correspondence with another set of points (the sets correspond, not the individual points) [2].

As the problem has been formulated over the years one camera is used, and so the number of three dimensional motion parameters that have to be and can be computed is five: two for the direction of translation and three for the rotation.

In this paper we present a theory for the determination of three dimensional motion and structure from line correspondences in three views. A line is represented by its slope and intercept, and not by its endpoints, even if such points exist.

2. Motivation and previous work

The basic motivation for this research is the fact that optical flow (or discrete displacement) fields produced from real images by existing techniques are corrupted by noise and are partially incorrect. Most of the algorithms in the literature that use the retinal motion field to recover three dimensional motion, or are based on the correspondence of microfeatures, fail when the input (retinal motion) is noisy. Some algorithms work reasonably well for images in a specific domain.

Some researchers [8] [22] [19] have developed sets of nonlinear equations with the three dimensional motion parameters as unknowns, which are solved by initial guessing and iteration. These methods are very sensitive to noise, as reported in [22] [8] On the other hand, other researchers [18] have developed methods that do not require the solution of nonlinear systems, but only of

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linear ones. Despite this, in the presence of noise the results are not satisfactory [18].

Prazdny, Rieger and Lawton presented methods based on the separation of the optical flow field into translational and rotational components, under different assumptions. [23] [24] But difficulties are reported with the approach of Prazdny in the presence of noise [12], while the methods of Rieger and Lawton require the presence of occluding boundaries in the scene, something that cannot be guaranteed *a priori*. Finally, Ullman in his pioneering work [29] presented a local analysis, but his approach seems to be sensitive to noise, because of its local nature.

Several other authors [17] use the optic flow field and its first and second spatial derivatives at corresponding points to obtain the motion parameters. But these derivatives seem to be unreliable when noise is present, and there is no known algorithm that can determine them reliably in real images.

At this point it is worth noting that all the aforementioned methods assume an unrestricted motion (translation and rotation). In the case of restricted motion (translation only) some robust algorithms have been reported [14]. All in all, most of the methods presented up to now for the computation of three dimensional motion depend on the value of flow or retinal displacements. Certainly, there does not yet exist an algorithm that can compute retinal motion reasonably (for example with 5% accuracy) in real images. [30]

Even if we had some way, however, to compute retinal motion acceptably, say with at most an error of 10%, we believe that all the algorithms proposed to date that use retinal motion as input (and one camera) would still produce non-robust results. The reason is that the motion constraint (i.e. the relation between three dimensional motion and retinal displacements) is very sensitive to small perturbations [27].

The third approach, that computes the motion parameters directly from the spatiotemporal derivatives of the image intensity function, gets rid of the correspondence problem and seems very promising. In [13] [10][20] the behavior of these methods with respect to noise is not discussed. Of course research on this topic is still at an early stage, but recent results [11][21] as well as ongoing work [25] indicate the potential of the approach. So, as the structure from motion problem has been formulated (for a monocular observer), it seems to be very difficult.

A possible solution to this difficulty is as follows: Instead of using correspondences between microfeatures such as points, why not try to use correspondences of macrofeatures? In this case, on the one hand the retinal correspondence process will be much easier, greatly reducing false matches, and on the other hand the constraint that relates three dimensional motion to retinal motion will be different and perhaps not as sensitive to small perturbations resulting from discretization effects. As macrofeatures, we can use lines or contours, since they appear in a rich variety of natural images. The contour based approach has been examined in [2]. Research on

the problem of motion interpretation based on line correspondences has been carried out by T.S. Huang and his colleagues [16][15] There, the problem of three dimensional motion computation has been successfully addressed in the restricted cases of only rotational or only translational motion. In the case of unrestricted rigid motion some good results have been obtained in [15], but the solution is obtained iteratively from a system of non-linear equations, and convergence of the solution to a unique value is not guaranteed if the initial value that is fed to the iterative procedure is not close to the actual solution.

3. Statement of the problem

The problem we are addressing is to compute the 3-D motion and structure of a rigid object from its successive perspective projections. Since the structure can easily be computed when the motion is known, we will first derive the equation of a 3-D line given the motion parameters and the images of the line in two successive frames. Then, using this, we show how to recover 3-D motion from line correspondences.

The imaging geometry is the usual one: The system $OXYZ$ is the object space coordinate system with the image plane perpendicular to the optical axis (z axis) at the point $o = [0,0,1]^T$, the focal length being 1. Let ox, oy be the axes of the naturally induced coordinate system on the image plane ($ox // OX, oy // OY$). The focal point (nodal point of the eye) is O and so an object point $[X, Y, Z]^T$ is projected onto the point $[x, y]^T$ on the image plane, where

$$x = \frac{X}{Z}, \quad y = \frac{Y}{Z} \quad (1)$$

Finding structure is equivalent to finding the equations of all the 3-D lines of interest. These equations have the following form:

$$E_i: [X = A_{xi} Z + B_{xi}, \quad Y = A_{yi} Z + B_{yi}] \quad i = 1, 2, \dots$$

We use as motion parameters the rotation matrix R , representing a rotation around an axis that passes through the origin, and the translation vector T , where:

$$R \equiv \begin{bmatrix} r_1 & r_4 & r_7 \\ r_2 & r_5 & r_8 \\ r_3 & r_6 & r_9 \end{bmatrix}, \quad T \equiv \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

and r_1, r_2 etc as defined in [28] [26], $n_i, i = 1, 2, 3$ are the directional cosines of the rotation axis. A point $[X, Y, Z]^T$ before the motion is related to itself $[X', Y', Z']^T$ after the motion by

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = R \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + T$$

The above is enough to describe any rigid motion. The images are known 2-D lines of the form

$$\epsilon_a: \quad y = a_a x + b_a, \quad i = 1, 2, 3, \dots \quad l: a, b, c$$

Frames are denoted by letters. Also the lines ϵ_a and ϵ_a' correspond to the same line E_i in space.

We are also going to use another representation of the lines in vector form which, although dual to the equa-

tion form, can make the rotation computations look more natural. We represent an image line by the vector normal to the plane defined by the origin and the object line (which also contains the image line)

$$\epsilon_{ii} : y = a_{ii} x + b_{ii} \quad , \quad i = 1, 2, 3, \dots \quad l: a, b, c$$

or in vector form

$$\epsilon_{ii} : \begin{bmatrix} a_{ii} \\ -1 \\ b_{ii} \end{bmatrix}$$

We use a displacement and a direction vector to represent the object line:

$$E_i : \begin{aligned} X &= A_{zi} Z + B_{zi} \\ Y &= A_{yi} Z + B_{yi} \end{aligned}$$

or

$$d_i = \begin{bmatrix} A_{zi} \\ A_{yi} \\ 1 \end{bmatrix} \quad f_i = \begin{bmatrix} B_{zi} \\ B_{yi} \\ 0 \end{bmatrix}$$

and $E_i : f_i + Z d_i$

The first problem for which we propose a solution is that of finding structure from *motion and line correspondence*, i.e. finding the equation, before the motion, of a line in 3-D given the equations of two successive images of it, as well as its motion parameters R, T . We first consider the case of no rotation and then we introduce rotation. The second problem that we will solve is that of finding the motion and structure knowing only the *line correspondences* over three frames. If we then solve the first problem twice, first for frames 1 and 2 and then for frames 1 and 3, clearly we should obtain the same line representation. This is the only constraint on the motion parameters of the problem, and is enough to solve it if we have an adequate number of line correspondences. (We need a minimum of 6, as pointed out in [16][15], and in order to have linear equations it seems that we need 13.)

4. Structure from motion and correspondence in the pure translation case

A line E :

$$\begin{aligned} X &= A_x Z + B_x \\ Y &= A_y Z + B_y \end{aligned}$$

when translated by $T: [t_x, t_y, t_z]^T$ becomes

$$\begin{aligned} X &= A_x Z + B_x + t_x - A_x t_z \\ Y &= A_y Z + B_y + t_y - A_y t_z \end{aligned} \quad (2)$$

The images are known to exist and are

$$e_a : y = a_a x + b_a, \quad e_b : y = a_b x + b_b \quad (3),(4)$$

for the two frames respectively. From the imaging geometry we find from (2) and the relations of perspective projection (1), by eliminating X, Y, Z , that

$$y = x \frac{B_y}{B_x} - \frac{A_x B_y + A_y B_x}{B_y} \quad (5)$$

$$y = x \frac{B_y + t_y - A_y t_z}{B_x + t_x - A_x t_z} - \frac{A_x (B_y + t_y - A_y t_z) + A_y (B_x + t_x - A_x t_z)}{(B_y + t_y - A_y t_z)} \quad (6)$$

By equating the x, y coefficients of (3-5) and (4-6) we get four equations in four unknowns (the parameters of the 3-D lines). Solving them we get only two solutions (one is spurious [26]). The valid one can be written in vector form :

$$d = \frac{\epsilon_a \times \epsilon_b}{\bar{Z} \cdot (\epsilon_a \times \epsilon_b)}, \quad f = \frac{(T \cdot \epsilon_b) (\bar{Z} \times \epsilon_a)}{\bar{Z} \cdot (\epsilon_a \times \epsilon_b)} \quad (7)(8)$$

where \bar{Z} is the unit vector along the z axis.

5. Introducing rotation

The general case with both rotation and translation can be derived directly from the pure translation case quite easily. We first establish the following result which is also used in [16][15]:

An image line ϵ_a (in vector form) of a line in space that is rotating with rotation R around the origin, is transformed into an image line $R \cdot \epsilon_a$.

PROOF: See [16][26]

The importance of the above result is that the rotated image can be found without any knowledge about the object line, which implies that no constraint can be derived from the pure rotation case to lead to a solution similar to that in the pure translational case. So we consider now the general case of both rotation and translation.

The movement of the line consists of a rotation followed by a translation. So if we rotate the first image ϵ_a to $R \cdot \epsilon_a$ then we can solve the pure translation case with the image of the first frame being $R \cdot \epsilon_a$ and the image of the second being ϵ_b , and what we get is the object line rotated by R . All we need then is to rotate back by R^T . This way equations (7), (8) become

$$d = \frac{\epsilon_a \times (R^T \cdot \epsilon_b)}{\bar{Z} \cdot (\epsilon_a \times (R^T \cdot \epsilon_b))}, \quad f = \frac{(T \cdot \epsilon_b) (\bar{Z} \times \epsilon_a)}{\bar{Z} \cdot (\epsilon_a \times (R^T \cdot \epsilon_b))} \quad (9)(10)$$

In the above expressions the z components of the vectors are 1 and 0 respectively. This not only makes the duality of the vector and equation forms obvious but it is also a sufficient property to guarantee that two equal lines are always represented by the same pair of vectors, a fact that we use in the next section.

6. Motion and structure from line correspondences

In the previous section we showed how to compute the structure given the line correspondences and the motion. Here we are concerned with finding motion from line correspondences alone.

Given the images of one line in three successive frames (a, b, c) , the solution (as a function of the R and T parameters) must be the same for both pairs of frames $b-c$ and $a-b$. So

$$d = \frac{\epsilon_a \times (R_a^T \cdot \epsilon_b)}{\bar{Z} \cdot (\epsilon_a \times (R_a^T \cdot \epsilon_b))} = \frac{\epsilon_a \times (R_b^T \cdot \epsilon_c)}{\bar{Z} \cdot (\epsilon_a \times (R_b^T \cdot \epsilon_c))} \quad (11)$$

$$f = \frac{(T_a \cdot \epsilon_b) (\bar{Z} \times \epsilon_a)}{\bar{Z} \cdot (\epsilon_a \times (R_a^T \cdot \epsilon_b))} = \frac{(T_b \cdot \epsilon_c) (\bar{Z} \times \epsilon_a)}{\bar{Z} \cdot (\epsilon_a \times (R_b^T \cdot \epsilon_c))} \quad (12)$$

where ϵ_c is the image of the line in the third frame and

T_a, T_b, R_a, R_b represent the translation and rotation for frames $a-b$ and frames $a-c$ respectively. We now simplify these vector equations since they represent four equations, only two of which are independent. (The proof is omitted here; instead an intuitive explanation is given.) The vector f represents the point where the line cuts the plane $Z = 0$. This point belongs to this plane and to the plane defined by the origin and the image line, which of course contains the object line, so it belongs to their intersection which we can find from the image alone. Thus given the x (y) component of the f vector the y (x) component can be found. This implies that another equation in f is superfluous. For the d vector we know that it has $x = 1$ and is orthogonal to the image line vector. The only additional information we need to specify it is the one of the other two components. The third can be found then. So we can have only one independent equation in the d vector.

Equations (11), (12) can be expanded and from them we choose the ones that come from equating the x components of the vectors. There is no reason for this, other than the fact that they lead to simpler equations and are independent. We can write them also as

$$\frac{(T_a \cdot \epsilon_b)(\epsilon_a \cdot \bar{Y})}{(\epsilon_a, R_a^T \epsilon_b, \bar{Z})} = \frac{(T_b \cdot \epsilon_c)(\epsilon_a \cdot \bar{Y})}{(\epsilon_a, R_b^T \epsilon_c, \bar{Z})} \quad (13)$$

$$\frac{(T_a \cdot \epsilon_b)(\epsilon_a \cdot \bar{Y})}{(\epsilon_a, R_a^T \epsilon_b, \bar{X})} = \frac{(T_b \cdot \epsilon_c)(\epsilon_a \cdot \bar{Y})}{(\epsilon_a, R_b^T \epsilon_c, \bar{X})} \quad (14)$$

where (\cdot, \cdot, \cdot) is the scalar triple product of vectors. By simplifying the triple products and cross multiplying and then substituting

$$K = (T_b \cdot R_{a1})^T - T_a \cdot R_{b1}$$

$$L = (T_b \cdot R_{a2})^T - T_a \cdot R_{b2}$$

$$M = (T_b \cdot R_{a3})^T - T_a \cdot R_{b3}$$

where R_{a1} is the first column of the matrix R_a etc we get

$$a_a (\epsilon_b^T \cdot L \cdot \epsilon_c) + (\epsilon_b^T \cdot K \cdot \epsilon_c) = 0 \quad (15)$$

$$b_b (\epsilon_b^T \cdot L \cdot \epsilon_c) - (\epsilon_b^T \cdot M \cdot \epsilon_c) = 0 \quad (16)$$

from eqs. (13), (14) respectively. The above equations are non-linear in terms of the motion parameters but linear in terms of the elements of the matrices K, L, M , and they come from considering just one line. By using 13 lines we can get 26 linear equations, set any of the 27 elements of the matrices to 1 and solve the 26×26 system; then we can find the elements of the K, L, M matrices which in terms of the motion parameters are:

$$K = \begin{bmatrix} r_{a1} t_{bz} - r_{b1} t_{az} & r_{a1} t_{by} - r_{b4} t_{az} & r_{a1} t_{bz} - r_{b7} t_{az} \\ r_{a4} t_{bz} - r_{b1} t_{ay} & r_{a4} t_{by} - r_{b4} t_{ay} & r_{a4} t_{bz} - r_{b7} t_{ay} \\ r_{a7} t_{bz} - r_{b1} t_{ax} & r_{a7} t_{by} - r_{b4} t_{ax} & r_{a7} t_{bz} - r_{b7} t_{ax} \end{bmatrix}$$

and similarly L, M .

In this way it is easy to find the numerical values of the three matrices. By equating their values with the functions of the motion parameters that they represent we get 27 nonlinear equations involving the motion parameters only. By setting one of the values to 1 we actually set the scale factor of the solution to some value.

7. Solving for the motion parameters

Now what we have to do is solve for the motion parameters, given that we know the K, L, M matrices. The procedure to find them is the following: first find the direction of the translation and the directions of the column vectors of the rotation matrices, and then the magnitude of the translation and the polarities of the rotation columns. The second part needs more explanation. It is well known that this family of problems has an inherent ambiguity in the estimation of the translations and absolute positions. These can be found up to a scale factor only and there is nothing that we can do about this. But the magnitude of the translation we compute does not represent anything more than the arbitrary choice of the 27th of the elements of the matrices to be unity. The only thing we need these magnitudes for is their ratio, which is valid since the common scale factor is eliminated. For the rotation columns we don't need to find their magnitude, since it is 1, but we have to find their polarity, which can be found easily.

The three matrices can be written as

$$K = \begin{bmatrix} t_{az} & r_{a1} \\ t_{ay} & r_{a4} \\ t_{ax} & r_{a7} \end{bmatrix} \cdot \begin{bmatrix} -r_{b1} & -r_{b4} & -r_{b7} \\ t_{bz} & t_{by} & t_{bz} \end{bmatrix} \quad (17)$$

and similarly L, M . The eigenvector that corresponds to the eigenvalue zero of the matrix M must be orthogonal to the T_b and to R_{b1} , and the same holds for the other two of them. If we consider the transpose of the matrix M , then the eigenvector is orthogonal to T_a and to R_{a1} . Let these six vectors be $f_1, f_2, f_3, f'_1, f'_2, f'_3$, for the vectors of the a movement and the b movement and the three matrices respectively. The cross product of any two of them, if they are not collinear, yields the direction of the corresponding translation. The following theorem provides the conditions for this.

The direction of translation 'a' can be estimated from the cross product of two of the f 's when the following hold:

a) R_{b1} and T_b are linearly independent

b) the analog of condition a) with circular substitution of 1, 2 and 3

PROOF: For proof and discussion see [26].

The vectors f_1, f_2, f_3 provide sufficient constraints for the recovery of the rotation column vectors too. The problem for this recovery can be stated as follows:

Given three vectors f_1, f_2, f_3 , find three pairwise orthogonal vectors r_1, r_2, r_3 , such that f_i is orthogonal to r_i for $i=1, 2, 3$.

This problem has two solutions in general. Before we present a way to find them we try to give a more visual description of the problem. The vectors f_1, f_2, f_3 , define three planes that meet along a line parallel to the translation vector. Obviously each of r_1, r_2, r_3 belongs to each of these planes. So the problem is equivalent to fitting an orthogonal system into three planes that meet along a line. In order to find the solution it is enough to find the solution for r_1 , because r_2 is orthogonal to r_1 and f_2 , so is parallel to their cross product, and its length is

known to be unity. The same holds true for the third vector. In order to find r_1 we define a vector k that is supposed to be orthogonal to r_1 ; the only constraint we have is that

$$((k \times f_1) \times f_2) \cdot ((k \times f_1) \times f_3) = 0$$

The above scalar equation states all the necessary conditions for the problem. There are infinitely many solutions to this equation and all the nonzero ones are equivalent for our purpose. To find just one we can arbitrarily set two of the components of the k vector to any convenient values. We choose the y, z components to be 1, 0, respectively, because these values are simple and do not, in general, lead to a degenerate solution. If such a solution is detected (the cross product of f_1 and k should then be zero) the reason might be that the values chosen were bad or that the problem itself is ambiguous. (One such case of ambiguity is when f_1, f_2 are parallel and f_3 is orthogonal to them, and similarly for all its cyclically symmetrical cases.) The first of the two cases is easily detected and we can then repeat the process by choosing better arbitrary components for the k vector, ones that can not lead to k vectors parallel to f_1 . After the substitutions and the simplifications are done the equation we get is

$$\alpha k_x^2 + \beta k_x + \gamma = 0 \quad (18)$$

where α, β, γ are functions of the components of the f_1, f_2 etc [26]. Equation (18) has two solutions. These give two values of the k vector that lead to two sets of directions for the column vectors of the matrix r . We already know that these columns have length 1 so what remains to be found is their orientations along their axes. The method we present finds one valid set of orientation for only one value of the k vector (the rest of the orientations or values are rejected because either they are not compatible with the initial equations or they lead to rotation matrices with negative determinants). One of the two solutions for the rotation matrix turns out to be spurious without any physical interpretation and incompatible with the initial equations, but we haven't yet established why there should be only one solution. Yet the spurious one is easily identified because it leads to an inconsistent linear system. The magnitude of the translation is computed as follows:

Let T'_a be the unit vector representing the direction of the translation a and T'_b the unit vector representing the direction of the translation b . Let ρ_{ii} and σ_{ii} be the corresponding directions of the translations and $R'_{ai}, R'_{bi}, \sigma_{ri}$ and ρ_{ri} the directions and the polarities of the rotation columns i (notice the opposite order) which take on only +1 or -1 as their values. If we do the substitutions ($\rho_{ii} T'_{ai}$ for T_{ai} , etc.) we get that K is equal to

$$\begin{bmatrix} \rho_1 r'_{a1} t'_{bz} - \sigma_1 r'_{b1} t'_{az} & \rho_1 r'_{a1} t'_{by} - \sigma_1 r'_{b4} t'_{az} & \rho_1 r'_{a1} t'_{bz} - \sigma_1 r'_{b7} t'_{az} \\ \rho_1 r'_{a4} t'_{bz} - \sigma_1 r'_{b1} t'_{ay} & \rho_1 r'_{a4} t'_{by} - \sigma_1 r'_{b4} t'_{ay} & \rho_1 r'_{a4} t'_{bz} - \sigma_1 r'_{b7} t'_{ay} \\ \rho_1 r'_{a7} t'_{bz} - \sigma_1 r'_{b1} t'_{az} & \rho_1 r'_{a7} t'_{by} - \sigma_1 r'_{b4} t'_{az} & \rho_1 r'_{a7} t'_{bz} - \sigma_1 r'_{b7} t'_{az} \end{bmatrix}$$

and similarly for L, M . where $\rho_i = \rho_i, \rho_{r,1}$, and the same for the rest. The equations above are three systems of linear equations that have more equations than unknowns and so it is easy to check for incompatibility of the spuri-

ous solution. For the other solution we will get a unique value for ρ_1, ρ_2, ρ_3 , from which we can infer that $\rho_i = +\rho_1$ or $\rho_i = -\rho_1$ since ρ_{ri} is either +1 or -1. It is clear that there are two sets of signs that satisfy the above constraints. One corresponds to a left and the other to a right handed coordinate system, only the second being of interest to us. In order to check which one is left-handed we form the rotation matrices and find the determinants and keep the solution that gives the positive determinant. This is the only solution that we get and our simulations show that indeed it is the correct one.

8. Experiments

We have done several experiments using randomly generated lines and motion parameters. The results were very accurate in the absence of noise. Due to lack of space the results are not reported here. They can be found in [26]. In case of noise the results are affected. We are currently doing systematic experiments and working on the development of a mathematical theory of the stability of the algorithm.

9. Conclusions

We have presented a method for computing structure and motion from line correspondences. The method briefly is as follows: Extract 13 lines from the image, approximate their equations, and then form a 26×26 matrix to find the elements of the K, L, M matrices. Some preliminary experiments indicate sensitivity to noisy input (by noisy here we mean inaccurate parameters of the image lines, not bad correspondence, since the possibility of the latter type of error is very small). The sensitivity of the solution of the linear system seems to be very high and might cancel the advantage we get by using lines (the parameters of which can be computed with better accuracy than in the case of points). The model for the noise we used wasn't good enough to permit comparison of the point and line correspondence methods.

It is worth noting that the method gives a unique solution in general unless the lines we choose result in a system with determinant very close to 0, as became evident from the experiments. We are working towards establishing both experimental and theoretical results on the stability of the proposed algorithm and conditions for the uniqueness of the solution.

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