MAX-MIN CHAINING OF WEIGHTED CAUSAL ASSERTIONS IS LOOP FREE

S. W. Ng and Adrian Walker

Work performed at Rutgers University*

ABSTRACT

If a system uses assertions of the general form *x causes y* , (e.g. MYCIN rules) then loop situations in which X_1 *causes* X_2, X_2 *causes* X_3, ... , X_n *causes* X_1, are, intuitively, best avoided. If an assertion has an attached confidence weight, as in *x (0.8)-causes y* , then one can choose to say that the confidence in a chain of such assertions is as strong as the weakest link in the chain. If there are several chains of assertions from X to Z, then one can choose to say that X causes Z with a confidence equal to that of the strongest chain.

From these choices, it follows that the confidence that X causes Z corresponds to a loop-free chain of assertions. This is true even if there are chains from X to Z with common subchains and loops within loops. An algorithm for computing the confidence is described.

I INTRODUCTION and TERMINOLOGY

There is currently considerable interest in representing knowledge about a practical situation in the form of weighted cause-effect or situation-action rules, and in using the knowledge so represented in decision-making systems. For example, in medical decision making systems, the rules may represent causal trends in a disease process in a patient [6], or the rules may represent trends in the decision process of a physician who is diagnosing and treating a patient [2,4]. In such representations, the chaining together of rules can be written as a weighted, directed graph. In MYCIN [2] the graphs are like *and-or* trees, while in OCKHAM [3,4,5] the graphs may have loops. This paper presents a result which appears in [1]. From the result it follows that, using the max and min operations, a graph containing loops can be interpreted as though it were loop-free.

*Authors' present addresses:
S. W. Ng, 6F Wing Hing Street, Hong Kong.
Adrian Walker, Bell Laboratories, Murray Hill, NJ.

The kind of graph in question is a *stochastic graph* (sg) consisting of *nodes* $N = \{1,2,...,n\}$ and a function P from $N \times N$ to the real numbers w, $0 \le w \le 1$. P is such that, for each $i \in N$, $\sum_{j=1}^{n} P(i,j) \le 1$. If $P(i,j) = w$, then w is the *weight* of the arc from node i to node j. A *path* in an sg is a string $n_1 \cdots n_l \in N^+$ such that $P(n_k, n_{k+1}) > 0$ for $1 \le k < l$. $n_2,...,n_{l-1}$ are *intermediate nodes* of $n_1 \cdots n_l$. A path $n_1 \cdots n_l$ of a graph is said to have a *loop* if $n_i = n_j$ for some i,j such that either $1 \le i < j < l$ or $1 < i < j \le l$. Otherwise the path is *loop-free*. The *weight of a path* $n_1 n_2 \cdots n_l$ of an sg is the minimum over $1 \le i < l$ of the weight of the arc from n_i to n_{i+1}. The *k-weight* w_{ij}^k from node i to node j of a graph, is the maximum of the weights of all the paths from i to j having no intermediate node with number higher than k. The *weight* w_{ij} from node i to node j of an sg is w_{ij}^n.

II EXAMPLES

This section gives examples of potential causal loops, in MYCIN [2] and in OCKHAM [3,4,5], and it shows how these loops are avoided by the use of the maximum and minimum operations.

A. A MYCIN Example

Consider a set of MYCIN rules

$B \wedge C \ (1.0) \rightarrow A$

$B \ (1.0) \rightarrow D$

$D \vee E \ (0.5) \rightarrow B$

$G \wedge H \ (0.5) \rightarrow B$

and suppose that C, E, G, and H are known with confidences 0.9, 0.8, 0.5, 0.4, respectively. Writing $c(X)$ for the confidence in X, confidences propagate through rules by:

$c(Z) = w \, . \, max(c(X), c(Y))$

for

$X \vee Y \; (w) \rightarrow Z$

and,

$c(Z) = w \, . \, min(c(X), c(Y))$

for

$X \wedge Y \; (w) \rightarrow Z.$

The greatest confidence which can be computed in A is $c(A) = 0.4$ by the tree

$$A \leftarrow ((B \leftarrow (D \leftarrow (B \leftarrow G \wedge H)) \vee E) \wedge C)$$

B occurs twice, so the tree can be thought of as a graph with a loop. However, the value of $c(A)$ depends only on the loop-free path EBA.

B. An OCKHAM Example

The following set of OCKHAM [3,4,5] rules is intended to show a strategy of a person who is deciding whether to stay at home, go to the office, or to try for a standby flight to go on vacation. The external factors *project deadline, snowstorm, project completed,* and *another flight* influence the choice by placing the arc(s) so labelled in a stochastic graph. The rules are:

HOME (*project deadline*, 1.0)→ *OFFICE*

OFFICE (*snowstorm*, 0.5)→ *HOME*

OFFICE (*project completed*, 0.5)→ *AIRPORT_STANDBY*

AIRPORT_STANDBY (*another flight*, 0.25)→ *AIRPORT_STANDBY*

AIRPORT_STANDBY (*snowstorm*, 0.75)→ *HOME*

These rules make up a stochastic graph with nodes *HOME*, *OFFICE*, and *AIRPORT_STANDBY*. If all of the external factors *project deadline, snowstorm, project completed,* and *another flight* are true, then the graph has five arcs and multiple loops. If the weight from *HOME* to *AIRPORT_STANDBY* is considered, then it turns out to be 0.5. The corresponding path, *HOME−OFFICE−AIRPORT_STANDBY*, is loop-free.

III ALGORITHM and RESULTS

The algorithm MAXMIN, shown below, computes the weight from a given node to another node in an sg. Note that, by Step 2, MAXMIN runs in $O(n^3)$ time.

MAXMIN

Input: A stochastic graph of n nodes

Output: n^2 real numbers

Step 1: for $1 \le i,j \le n$ do $B_{ij}^0 := P(i,j)$

Step 2: for k:=1 to n do

for $1 \le i,j \le n$ do

$B_{ij}^k := max(B_{ij}^{k-1}, min(B_{ik}^{k-1}, B_{kj}^{k-1}))$

Step 3: for $1 \le i,j \le n$ do output B_{ij}^n

The properties of paths, path weights, and the values B_{ij}^k, described in the Lemma below, are established in Appendix I.

Lemma In an sg of n nodes, the following statements hold for $1 \le i,j \le n$ and for $0 \le k \le n$:

(i) If $w_{ij}^k > 0$, then there exists a loop-free path from i to j whose weight is w_{ij}^k,

(ii) $B_{ij}^k = w_{ij}^k$.

Setting k=n in parts (i) and (ii) of the Lemma yields the two results:

Result 1 In any sg the weight w_{ij}, that is, the maximum path weight over all paths from i to j, is equal the maximum over only the loop-free paths from i to j.

Result 2 If MAXMIN receives as input an sg with n nodes, then, for $1 \le i,j \le n$, the output B_{ij}^n is equal the weight w_{ij} from node i to node j of the graph.

Result 1 establishes a property of any sg, namely that the weight from one node to another is the weight of some loop-free path, while Result 2 establishes that MAXMIN is one possible algorithm for finding such weights.

IV CONCLUSIONS

In a system in which weighted causal assertions can be combined into causal paths and graphs, causal loops can occur. Common sense about everyday causality suggests that such loops are best avoided. If the weight of a path is chosen to be the minimum of the individual arc weights, and the net effect of a start node on a final node is chosen to be the maximum of the path weights from the start node to the final node, then the weights (by whatever algorithm they are computed) are independent of the presence of loops in the underlying graph. There is a simple $O(n^3)$ algorithm to compute these weights.

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APPENDIX I

Proof of Lemma Let $k=0$. If $w_{ij}^0 > 0$, then by definition, there exists a path from i to j having no intermediate nodes, whose weight is w_{ij}^0. Clearly this path is ij, which is loop-free. So we may write $\gamma_{ij}^0 = ij$, where γ_{ij}^k denotes a path from i to j having no intermediate node with number greater than k. Then $w_{ij}^0 = P(i,j) = B_{ij}^0$. If $w_{ij}^0 = 0$, then there is no such path, and $B_{ij}^0 = 0$.

Suppose, by way of inductive hypothesis, that for $1 \le i,j \le n$ and for some $(k-1) < n$,

(i) if $w_{ij}^{k-1} > 0$ then there is a loop-free path γ_{ij}^{k-1}, from i to j with each intermediate node at most $k-1$, whose weight is w_{ij}^{k-1}, and

(ii) $B_{ij}^{k-1} = w_{ij}^{k-1}$.

If $w_{ij}^{k-1} > 0$ then there is a path γ from i to j whose weight is w_{ij}^k. γ is such that either

(A) each intermediate node of γ is at most $(k-1)$, or

(B) γ goes from i to k, from k to k some number of times, then from k to j, with each intermediate node of each subpath being at most $(k-1)$.

This is because (A) and (B) exhaust all possibilities.

In case (A) it is clear that $w_{ij}^k = w_{ij}^{k-1}$, and the inductive step for part (i) of the Lemma is completed with $\gamma_{ij}^k = \gamma_{ij}^{k-1}$. In case (B), it follows from our induction hypothesis that there exist loop-free paths γ_{ik}^{k-1}, γ_{kk}^{k-1}, γ_{kj}^{k-1} with weights w_{ik}^{k-1}, w_{kk}^{k-1}, w_{kj}^{k-1} respectively. Let $w = min(w_{ik}^{k-1}, w_{kj}^{k-1})$ and $w' = w_{kk}^{k-1}$, and consider the subcases (B1) in which γ goes from k to k zero times, and (B2) in which γ goes from k to k one or more times. In (B1) the weight of γ is clearly w, while in (B2) it is $min(w, w')$. Hence, from the definition of w_{ij}^k, we have $w_{ij}^k = max(w, min(w, w'))$, which is simply w. So part (i) of the Lemma holds with $\gamma_{ij}^k = \gamma_{ik}^{k-1}\gamma_{kj}^{k-1}$. From part (ii) of the inductive hypothesis, and from Step 2 of the MAX-MIN algorithm, it follows that $B_{ij}^k = max(w_{ij}^{k-1}, w)$. So $B_{ij}^k = max(w_{ij}^{k-1}, w_{ij}^k) = w_{ij}^k$, since it follows from the definition of w_{ij}^k that $w_{ij}^k \ge w_{ij}^{k-1}$. So in either of the cases (A) and (B) $B_{ij}^k = w_{ij}^k$, which establishes part (ii) of the Lemma for the case $w_{ij}^k > 0$.

If $w_{ij}^k = 0$ then there is no path from i to j. Suppose $B_{ij}^k \ne 0$. Then either $w_{ij}^{k-1} \ne 0$, or both of w_{ik}^{k-1}, w_{kj}^{k-1} are nonzero. In each case there is a path from i to j, a contradiction. So if $w_{ij}^k = 0$ then $B_{ij}^k = w_{ij}^k$. □

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